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The Cosmogonical Significance of Stellar Rotation*

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I. The Problem

About ten years ago a number of astronomers made a concerted effort to study the axial rotations of the stars by means of their spectra. This effort led to some very interesting results concerning the distribution of rotational velocities among stars of different spectral characteristics. But at that time relatively little was known about the manner in which a spectral absorption line is produced in the atmosphere of a star. It was, therefore, not certain that the broadening of spectral lines should be attributed to rotation. The resulting skepticism, justified by these conditions, was expressed very succinctly by E. A. Milne[1] at a meeting of the Royal Astronomical Society in June, 1931: "I think that axial rotations of stars deduced from line contours are open to grave suspicion until we have a satisfactory theory of contours of absorption lines. The principle seems to be that anything that is not understood in the way of contour distortion is attributed to rotation. Presumably the method is one of the few ways we have of measuring the axial rotation of stars, but until the method has been successfully applied to the sun, we should, I think, receive the results with caution." However, in the intervening years stellar spectroscopy has made enormous advances. We now possess an excellent theory of the formation of stellar absorption lines and we know how to distinguish case of line broadening caused by other effects. The purpose of this paper is first to present a critical review of our present knowledge of stellar rotations and, if possible, to establish just how certain we are that the measured rotations are real. I shall then summarize the results of the observations in their relation to other branches of astrophysics and shall, in particular, try to answer the question: What becomes of a star when the rotation at the equator is excessive? Finally, I shall deal with what is probably the most fundamental problem of stellar rotation confronting us, namely: how have the rotational motions of the stars originated?

I believe it was Sir Arthur Eddington who, when told of the dis-

* A paper presented in February, 1942, at a symposium in connection with the dedication of the National Astrophysical Observatory of Mexico, in Tonanzintla, Puebla, Mexico.

[1] *The Observatory*, **54**, 9, 1941.

covery of large rotational velocities in many stars, said that the frequent occurrence in the universe of rotational motions constitutes one of the deepest mysteries of physical science. Of course, the concept of rotation is not new. It undoubtedly antedates that of translational motion, at least in so far as the universe is concerned. We might say that if astronomy is the oldest of the sciences then surely the concept of rotation constitutes the oldest result of scientific thinking. Thomas Wright of Durham had argued in the sixth letter of his famous work "An Original Theory and New Hypothesis of the Universe," published in 1750, that the stars all have attendant systems, similar to the solar system, and that they rotate around their axes as the sun does. In fact, he concluded that it is no more extraordinary to suppose that the stars move through space than it is to suppose that they rotate around their axes.[2] The origin of such axial rotation is not further discussed by Wright, and we next encouter it in the cosmogony of Kant: "The scattered elements (in the original nebula), by means of their attraction, gather from a sphere around them all the matter of less specific gravity . . . But nature has other forces in store, which are especially exerted when matter is decomposed into fine particles. They are those forces by which these particles repel each other, and which, by their conflict with attraction bring forth that movement, which is, as it were, the lasting life of nature. This force of repulsion is manifested in the elasticity of vapors. . . It is by it that the elements, which may be falling to the point attracting them, are turned sideways promiscuously from their movement in a straight line; and their perpendicular fall thereby issues in circular movements. . . Yet these movements are in many ways in conflict with each other, and they naturally tend to bring one another to uniformity, that is, into a state in which one movement is as little obstructive to the other as possible . . . till they are all moving horizontally, *i.e.*, in parallel circles round the sun as their center, no longer intersect each other, and by the centrifugal force becoming equal with the falling force they keep themselves constantly in free circular orbits at the distance at which they move.[3]

We recognize that Kant considers the creation of angular momentum in a system which initially had no such momentum. But Kant was not a mathematician, and the value of his idea must be sought in the plane of philosophy rather than that of formal science.

II. Historical Review

W. de W. Abney was the first astronomer to express, in 1877, the idea that the axial rotation of the stars could be determined from measurements of the widths of spectral lines: "There would be a total broadening of the line, consisting of a sort of double penumbra and a black

[2] W. Hastie, Kant's "Cosmogony," Glasgow, 1900.

[3] Hastie's translation of Kant's "Universal Natural History and Theory of the Heavens," Glasgow, 1900, p. 75-78.

nucleus. Supposing the surface of a quickly rotating star to be in the same condition as the sun . . . no absorption line due to the former can be as sharp and distinct as it is when seen in the solar spectrum."[4] This suggestion was severely criticised by H. C. Vogel who correctly pointed out that it was not sufficient to observe broad hydrogen lines but that all stellar absorption lines should be broadened if rotation were actually present. He incorrectly asserted, however, that "even in the case when all lines of a star spectrum should be appreciably broadened, one would not be justified in attributing this broadening to rotation, but one would be able to explain the phenomenon as a result of pressure and temperature in the atmosphere of the celestial object."[5] He also, rather unwisely, insisted that no star ever showed broadening in all lines, although he admitted that perhaps in the future "optical means might be perfected in a manner which is beyond our present comprehension." Incidentally, Vogel's paper contains the following statement in regard to the use of photography which had been suggested by Abney:

"It is quite impossible for me to understand how Mr. Abney can express the hope that the photography of stellar spectra could ever become of importance in the problem considered by him. For such faint objects as we generally find in the case of stellar spectra the method of photography will always be inferior to that of visual observation. Photography can be of importance only because it makes accessible the violet and ultraviolet parts of the spectrum, which are difficult to observe visually, and makes it possible for us to observe the stronger absorption lines in these regions. However, even on the most successful photograph only relative measures of lines would be possible in regard to their width so that no conclusions could be obtained in regard to rotation: the widths of lines on different photographic plates would depend upon the length of exposure, the sensitivity of the plate and the length of development." It is, indeed, a remarkable coincidence that only a few years afterwards Vogel himself introduced photography into stellar spectroscopy and completely overturned his pessimistic prediction of 1877. Nevertheless, the tremendous weight of his opinion undoubtedly retarded the application of Abney's interesting suggestion. Vogel computed that if the width of Hβ in α Lyrae were due to rotation the equatorial velocity would be 45 geographical miles. Similarly, for α Aquilae he concluded: "An equator point would have the very considerable velocity of 25 geographical miles . . . these velocities, especially when compared with that of the sun, namely 0.27 geographical miles, appear in the highest degree improbable." In a much later paper Vogel reversed his stand and expressed himself in favor of the hypothesis that rotation does produce the broadening of the lines of α Aquilae.[5a]

[4] *M. N.*, **37**, 278, 1877.
[5] *A. N.*, **90**, 74, 1877.
[5a] *Sitz. Berlin Ak.*, p. 46, 1898.

The small rotational velocity of the sun—2 km/sec—has discouraged other investigators. J. Evershed says in 1922: "The angular speed (of Sirius) will be over three times that of the sun, a complete rotation taking eight days or less. This high speed of rotation seems improbable considering that Sirius is in an earlier stage of evolution, and is therefore less condensed than the sun . . ."[6] Similarly J. A. Carroll expresses the opinion: "We are thus able to say that no stars have been found rotating with an equatorial velocity much greater than say 50 km/sec and that . . . the rotational speed must have been more nearly of the order of 10 km/sec or less."[7]

While these doubts were being expressed with regard to rotations in single stars Schlesinger had already in 1909 obtained convincing evidence of rotation in the eclipsing binary system δ Librae: "The rotation of the bright star has another consequence in certain parts of the orbit. In general, we obtain light from the whole disk and the observed velocity is equal to that of the center of the star. Just before and just after light minimum, however, this is not the case; before minimum the bright star is moving away from us and part of its disk is hidden by the dark star. The part that remains visible has on the whole an additional motion away from us on account of rotation. . . If we consult the table of normal places . . . we see that the observations accord with the above remarks. The residual for the first normal place is —2.9 km., and for the last +2.8 km. . . One indirect result of the presence of this effect has been to make us underestimate the accuracy of our elements."[8] A short time later Schlesinger announced a similar effect in λ Tauri.[9] The problem was pursued by G. Forbes,[10] and much later by Hellerich.[11] In 1924 R. A. Rossiter established the effect in β Lyrae and gave a complete curve of residuals in velocity during the eclipse,[12] while D. B. McLaughlin established the effect in Algol.[13] Since that time several other eclipsing systems have been investigated in this manner, especially by J. S. Plaskett, D. B. McLaughlin, and others. The latest announcement of the rotational effect in the radial velocities is for the eclipsing binary u Herculis.[14]

It is clear that if rotation is responsible for the displacements of the lines before and after light minimum, the contours must be unsymmetrical. This has been verified by Struve and Elvey in the case of Algol.[15] In fact, the observed contours agree very closely with those computed from the light-elements, assuming that the velocity of rota-

[6] *M. N.*, **82**, 395, 1922.
[7] *M. N.*, **88**, 555, 1928.
[8] *Pub. Allegheny Obs.*, **1**, 134, 1909.
[9] *Pub. Allegheny Obs.*, **3**, 28, 1910.
[10] *M. N.*, **71**, 719, 1911.
[11] *A. N.*, **216**, 277, 1922.
[12] *Ap. J.*, **60**, 15, 1924.
[13] *Ap. J.*, **60**, 22, 1924.
[14] Luyten, Struve, and Morgan, *Pub. Yerkes Obs.*, **7**, pt. 4, 24, 1939.
[15] *M. N.*, **91**, 663, 1931.

tion is 55 km/sec—this value having been found by McLaughlin from the period of orbital motion and the size of the principal star.

In ordinary close spectroscopic binaries, not known to be variable in light, Adams and Joy have for many years recognized that excessive broadening of the lines must be caused by rapid rotation in a system in which orbital and rotational periods coincide. For example, in W Ursae Majoris, of type F8, having a period of 0.334 day the lines are described as "so widened and weakened that measures of velocity and estimates of line-intensity can be made only with the greatest difficulty." The authors suggest that "the unusual character of the spectral lines is due mainly to the rotational effect in each star, which may cause a difference in velocity in the line of sight of as much as 240 km/sec between the two limbs of the star."[16]

A study of rotational line-broadening in spectroscopic binaries was made by Shajn and Struve.[17] The equatorial velocity of rotation in a close system is

$$v = r\omega \sin i = (2\pi r/P) \sin i \qquad (1)$$

But we have for sin i the following expression:

$$\sin i = \text{const.}\ K \cdot P^{1/3} (1 - e^2)^{1/2} [(m_1 + m_2)/m_2^{3/2}]^{2/3}$$

Selecting a group of stars for which

$$\left.\begin{array}{l} m_1 + m_2 = \text{const.} \\ m_2/m_1 = \text{const.} \\ (1 - e^2)^{1/2} = \text{const.} \end{array}\right\} \qquad (2)$$

we have

$$\sin i = K \cdot P^{1/3}$$

and from (1):

$$v = \text{const.}\ (r \cdot K/P^{2/3}) \qquad (3)$$

The conditions (2) will be approximately obeyed if we select stars according to spectral type and luminosity. This has been done for the B's and the A's. For each group K has been plotted as a function of P and the width of the line indicated by an appropriate symbol. (Fig. 1). In the diagrams the lines of constant rotational velocity are given by

$$K = \text{const.}\ P^{2/3} \qquad (4)$$

and these curves run slantwise through the diagrams from lower left to upper right. The observed rotational velocities quite closely follow these curves. This result justifies the conclusion that in spectroscopic binaries, at least, line broadening is essentially a result of rotation.

The next step was to extend the work to single stars. This was done, I believe, first in 1923, in connection with a study of the brighter stars in the Hyades.[18] The spectra of some of these stars have very diffuse lines, yet they do not otherwise differ very materially from stars whose lines are sharp. There was then suspected to be a small difference in

[16] *Ap. J.*, **49**, 189, 1919.

[17] *M. N.*, **89**, 222, 1929.

[18] O. Struve, Thesis, University of Chicago, 1923, unpublished.

luminosity between "n" and "s" stars—the well-known absolute magnitude criterion of Adams and Joy for stars of classes B and A. At any rate, I gave a number of fairly plausible reasons why the otherwise unexplained difference in line-width could best be explained as a result of rotation. In 1930 Elvey measured the contours of broad-lined stars, and after adopting the rotational interpretation gave the first list of actual rotational velocities, $v_0 \sin i$, derived from a graphical method suggested by Shajn and Struve.[19]

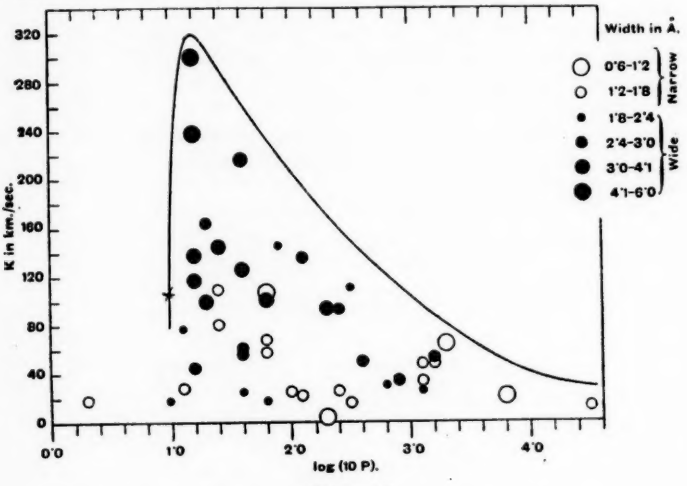

FIGURE 1

Line widths in spectroscopic binaries of types O and B. The two highest stars in the diagram are V Puppis and μ^1 Scorpii, for which Miss Maury finds a width of 5.8 Å and 4.9 Å, respectively.

The idea underlying this method was to assume that the undisturbed contour of a line is similar to that actually observed in a narrow-lined star of similar spectral class. For example, to find the rotational velocity of α Aquilae it was supposed that the undisturbed contour would be similar to that in α Lyrae or in α Cygni, or in Sirius. The "washed out" rotational contours were then computed by a simple graphical procedure.

Let x and y be measured on the apparent disk of the star. Then

$$y = \sin b$$
$$x = \cos b \sin l$$

The radial component of the rotational velocity is

$$v_1 = v_0 \cos b$$
$$v = v_1 \sin l = v_0 \cos b \sin l = v_0 x$$

Hence all points on the disk having equal x have also equal v. If the

[19] *Ap. J.*, **71**, 221, 1930.

axis is inclined all velocities must be multiplied by sin i.

Consider now the effect of rotation upon a line contour I $(\lambda - \lambda_0)$. This is observed from all parts of the disk:

$$J(\lambda) = \int_{-1}^{+1} \int_{0}^{y} I(\lambda - \lambda_0)\, dx\, dy$$

given

$$y = \sqrt{(1 - x^2)}$$

$$J(\lambda) = \int_{-1}^{+1} \int_{0}^{\sqrt{(1-x^2)}} I(\lambda - \lambda_0)\, dx\, dy$$

But

$$x = v/v_0;\ dx = (1/v_0)\, dv$$

Hence

$$J(\lambda) = (1/v_0) \int_{-1}^{+1} I(\lambda - \lambda_0)\ \sqrt{(1 - v^2)}\, dv$$

Here I $(\lambda - \lambda_0)$ is to be considered a function of v:

$$\lambda_0 = \lambda_1 + (\lambda_1 v/C)$$

by the Doppler effect. Accordingly, we have

$$J(\lambda) = \int_{-1}^{+1} I\{\lambda - \lambda[1 + (v/C)]\}\ \sqrt{(1 - v^2)}\, dv \qquad (5)$$

This expression can be integrated graphically if I $(\lambda - \lambda_0)$ is known.

In actual practice it is convenient to suppose that the apparent disk of the star is divided into a finite number of sections parallel to the axis of rotation. Each gives a contour

$$I = f(\lambda - \lambda_0)$$

where

$$\lambda_0 = \lambda_1 + \lambda_1 (v_0 x/r) \sin i$$

The area of each section is

$$A = 2 \int_{x_1}^{x_2} \sqrt{(r^2 - x^2)}\, dx = \left\{ x \sqrt{(r^2 - x^2)} + r^2 \arcsin (x/r) \right\}_{x_1}^{x_2}$$

Rotation shifts each contour by

$$[(x_1 + x_2)/2]\, v_0 \sin i$$

Hence we have the superposition of a number of contours:

$$J_n = f\{\lambda - [\lambda_1 + \lambda_1 (x_1 + x_2)\, v_0 \sin i/2r]\} A_n,$$

The final contour is

$$I = (1/\pi r^2)\, J_n,$$

where the factor πr^2 normalizes the result. Since the function f is obtained from observations the integration can best be obtained graphically. The procedure then consists in fitting the best contour I to the observed curve and in adopting the corresponding $v_0 \sin i$ for the rotational velocity.

This graphical procedure has been widely used by Elvey and also by me.[20] It was at first open to one important criticism. There was as yet no certainty that the "undisturbed" contour of a rapidly rotating star is actually similar to that of a slowly rotating star, even though the latter is chosen to have the same equivalent width.

This doubt has been removed by a very ingenious procedure suggested by Carroll.[21] The distorted contour contains in its shape certain features which originate from the original contour. The point here is that the disturbing contour is given essentially by the function $\sqrt{(1-v^2)}$ in (5) and is known, except for the constant v_0. Now, although v_0 is unknown, Carroll succeeded in showing how the original contour can be reconstructed and, in effect, how it and v_0 can be determined simultaneously. The method constitutes a remarkable piece of mathematical analysis. Carroll intended it to be used for the determination of v_0, and it has been used for this purpose by him and Miss Ingram, and also by Colacevich. However, Krat has criticized the procedure in several papers and has shown that errors of observations may easily result in wrong values of v_0. The fundamental service which Carroll has rendered is to show that in the case of a few well-observed rotational contours, such as that of Algol, the original undisturbed contour is very similar to that observed in slowly rotating stars. This justifies the procedure of Struve and Elvey.

In order to ascertain how much confidence we can place in the rotational origin of the dish-shaped contours measured by Elvey and others, we have to consider the following points:

1. The line widths of stars with very broad and shallow contours are proportional to the wave-lengths. This was first shown by Struve[22] and later confirmed by Shajn.[23] It strongly suggests that the broadening is produced by Doppler effect.

2. The relative intensities of members of multiplets have their normal values. For example, in the B stars the three SiIII lines 4553, 4568, 4575 have the relative intensities $\sqrt{5} : \sqrt{3} : 1$, while the multiplet rules give $5 : 3 : 1$. This agrees exactly with the relative intensities in normal main-sequence stars. Originally obtained by Struve and Elvey,[24] this result has been recently confirmed by I. H. Abdel-Rahman.[25] It will be recalled that in stars with excessive turbulence the line intensities are usually quite different. Hence, we may say with confidence that the Doppler effect is not one of turbulence, but is rather due to the mechanical superposition of contours from different parts of the disk. However, it should be noted that there appear to be unexplained complications in some curves of growth of turbulent stars of early type. Thus,

[20] *Ap. J.*, **72**, 1, 1930.
[21] *M. N.*, **93**, 478, 508, 680, 1933.
[22] *Ap. J.*, **72**, 8, 1930.
[23] *Zs. f. Ap.*, **6**, 176, 1932.
[24] *Ap. J.*, **72**, 267, 1930.
[25] *M. N.*, **101**, 312, 1941.

α Cygni gives a small slope imitating the square-root law, as Struve and Elvey pointed out some years ago and as Aller[26] and Greenstein[27] have recently confirmed.

3. In the eclipsing binaries, for example, Algol, the radial velocity effect of McLaughlin leads to precisely the same result as the Struve-Elvey procedure from the line contour observed outside of eclipse. There is no room in these stars for any broadening agency other than rotation.

4. In spectroscopic binaries the rotational velocities agree statistically with formula (3) and again leave room only for rotational broadening. Other mechanical causes, such as expansion, must be negligible.

5. The contours observed by Elvey agree exactly with those computed from formula (5). This eliminates all unsymmetrical causes of broadening, such as pulsation, and gives great weight to the rotational interpretation.

6. It is true that the hydrogen lines and some of the helium lines show departures from the pure rotational contours. These departures have sometimes been used as arguments against the rotational interpretation of these contours.[28] But these particular lines are broadened by Stark effect and their behavior is not only not opposed to the rotational hypothesis, but is, as Shajn has shown[29] in excellent agreement with it.

7. Shajn has pointed out that because of the proportionality of the Doppler effect with wave-length the contours of lines are not only broader, but also shallower when λ is large. He confirms this effect by stating that on his Simeis spectrograms the Fe lines of α Aquilae completely disappear in the visual region, although the resolving power of the instrument is more than ample for the purpose.

Taken together, the evidence is overwhelmingly in favor of the rotational interpretation of the broad lines in such stars as α Aquilae, η Ursae Majoris, etc. This does not mean that rotation necessarily accounts for the entire observed broadening. But since in the past ten years no new mechanisms have been discovered it is extremely unlikely that any unknown mechanisms should play more than a subordinate rôle.

III. Summary of Observational Data

1. The measured values of $v_0 \sin i$ range from 0 to about 250 or 300 km/sec. There are, however, a number of B stars and perhaps also A stars, in which the lines are so broad and shallow as not to be susceptible to accurate measurement. Such stars have not been listed in the papers by Elvey, Struve, or Westgate. The latter omitted a number of stars whose lines were so diffuse as to be hardly visible on

[26] *Ap. J.*, **95**, 73, 1942.
[27] Unpublished.
[28] Payne and Maulbetsch, *Harvard Circ.*, No. 364, 1931.
[29] *Zs. f. Ap.*, **6**, 189, 1932.

plates taken with the single-prism spectrograph of the Yerkes Observatory, having a dispersion of 30 A/mm at λ 4500. But Morgan, using very small dispersion, has recently shown that several of these stars have absorption lines which are considerably broader than those of η Ursae Majoris, for which Struve finds $v_0 \sin i = 200$ km/sec. The conclusion seems justified that rotational velocities may occasionally be as large as 300, 400, or even more km/sec.

2. The kinetic energy of rotation is

$$E = \tfrac{1}{2}\,\omega^2 \int r^2\,dm$$

Here ω is the angular velocity of the particle dm, and its kinetic energy is $\tfrac{1}{2}\,m\,r^2\,\omega^2$. The quantity $I = \int r^2\,dm$ is the moment of inertia about the axis of rotation. We introduce the quantity K, the radius of gyration, which is defined by

$$K^2 = \int r^2\,dm/M$$

Hence the kinetic energy of the rotating body is

$$E = \tfrac{1}{2}\,K^2\,\omega^2\,M$$

For a homogenous sphere $K^2 = 0.4\ r^2$, where r is its radius. For a centrally condensed sphere K^2 is smaller. We have

$$E = 0.2\ M\ r^2\ \omega^2 = 0.2\,M\,r^2\,(2\pi/T)^2 = 0.2\,M\,(2\pi r/T)^2,$$

but $2\pi r/T = v_0 =$ velocity of rotation:

$$E = 0.2\,M\,v_0^2.$$

Take a star which rotates with $v_0 = 100$ km/sec $= 10^7$ cm/sec and whose mass is

$$M = 5\,M\odot = 10^{34}\ \text{gr.}$$

Then

$$E = 0.2 \times 10^{34} \times 10^{14}\ \text{ergs} = 0.2 \times 10^{48}\ \text{ergs.}$$

Allowing for the central condensation we may estimate

$$E = 10^{48}\ \text{to}\ 10^{47}\ \text{ergs.}$$

This is to be compared with the energy of velocity of translation:

$$T = 0.5\,mv^2 = 0.5 \times 10^{34} \times v^2.$$

For a normal B or A star $v = 10$ km/sec $= 10^6$ cm/sec. Hence

$$T = 0.5 \times 10^{34} \times 10^{12} = 0.5 \times 10^{46}\ \text{ergs.}$$

These quantities are comparable to the astrophysical stores of energy. For example, for the sun we have:[30]

Radiant energy	2.83×10^{47} ergs
Translatory energy of atoms and electrons	26.9×10^{47} ergs
Energy of ionization and excitation	$< 26.9 \times 10^{47}$ ergs

3. The most obvious correlation between $v_0 \sin i$ and other physical parameters is with spectral type. This was recognized by Struve and Elvey[31] and later verified by Westgate and by Shajn. The O, B, A, and early F stars have frequently large rotations. The late F's rotate, on the average, much slower, and among stars of classes G and later rapid rotations occur only in close spectroscopic binaries.

[30] Eddington, *I.C.S.*, 289, 1930.
[31] *M. N.*, **91**, 673, 1931.

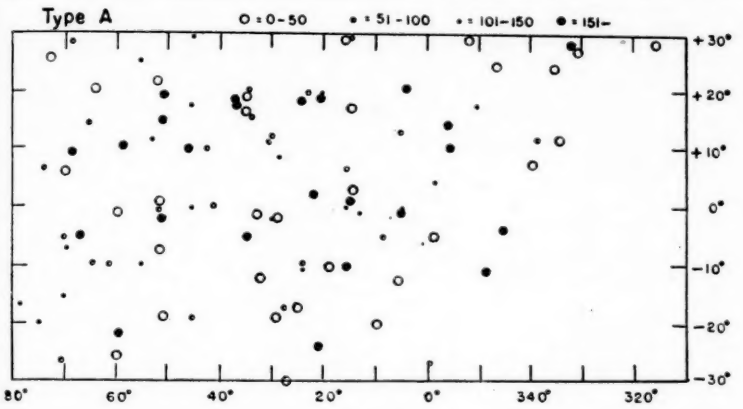

FIGURE 2

4. Supergiants of early or late type and normal giants of types F and later never show *conspicuous* rotations. This is true of Cepheids, long-period variables, and all other groups of stars which fall among the giants and supergiants. The conclusion is of rather recent origin. In our former work we felt reluctant to stress the absence of large rotations in the more luminous stars, because we thought that an early-type sugergiant, like β Orionis, if endowed with rapid rotation might not be recognized from its spectrum as a star of high luminosity. However, it now seems that criteria for identifying the supergiants are so complete that we are no longer left in doubt. There may, of course, exist moderate rotations among the supergiants which are masked by turbulence.

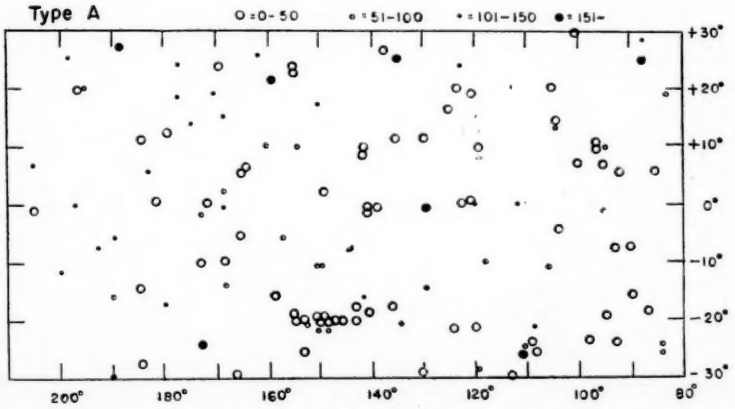

FIGURE 3

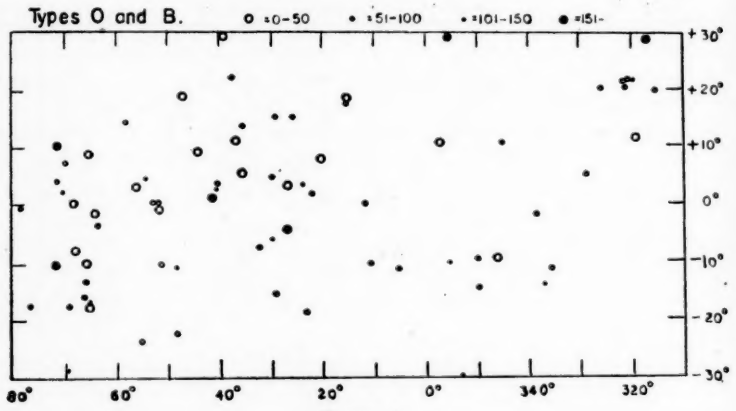

FIGURE 4

5. A plot of rotational velocity $v_0 \sin i$ according to galactic longitude and latitude brings out little information. Among the O and B stars there is no obvious relation with either longitude or latitude, but among the A's there is some tendency for the larger rotations to occur in longitudes 320° to 80°. In Figures 2, 3, 4, and 5 we notice several groups of stars which are somewhat distinguished either by uniformly large or by uniformly small rotational velocities. Thus, among the O and B stars there are pronounced groups of small rotations in Orion ($l = 175°$, $b = -20°$) and perhaps in Cassiopeia ($l = 90°$, $b = 0°$) and small groups of moderately large rotations in Perseus ($l = 115°$, $b = -8°$) and in Monoceros ($l = 185°$, $b = -8°$). Among the A's there is a rather striking assembly of small rotations in Taurus ($l = 150°$, $b =$

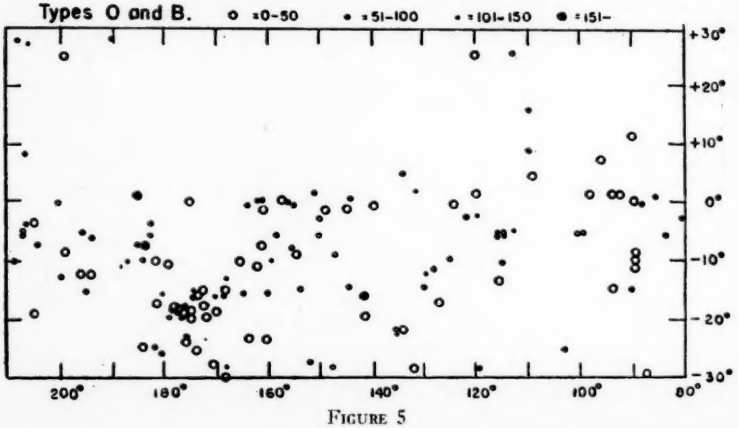

FIGURE 5

$-20°$) and a dispersed group of large rotations in Cygnus (around $l = 50°$, $b = +15°$).

We conclude that there are few regions in the sky where large rotations greatly predominate, and there is no reason to suspect that the axes of rotation are not essentially oriented at random.

6. TABLE 1 A shows that there is no appreciable correlation between

TABLE 1 A

MEAN RADIAL VELOCITIES
O AND B STARS

Rotational Velocities	Number of Stars	Mean Radial Vel.	Mean without Regard to Sign
0- 25	38	+10.3	16.9
26- 50	55	+ 4.8	16.8
51- 75	52	+ 6.1	17.9
76-100	70	+ 3.9	16.3
101-125	26	+ 5.1	15.6
126-150	18	+ 5.1	13.6
151-175	7	+ 5.1	11.1
176-200	3	−13.9	15.6
201-	2	+ 4.4	15.3
		A STARS	
0- 25	99	+ 0.1	14.1
26- 50	74	+ 0.7	14.1
51- 75	50	− 2.0	14.6
76-100	42	− 3.4	12.7
101-125	42	− 1.2	13.7
126-150	37	− 3.1	11.9
151-175	25	− 7.0	15.6
176-200	20	−10.5	12.9
201-	7	− 5.8	13.9

TABLE 1 B

MEAN RESIDUAL RADIAL VELOCITIES
TYPE O-B

Rotational Velocities	Number of Stars	Mean Radial Vel.	Mean without Regard to Sign
0- 25	38	+ 6.3	9.3
26- 50	55	+ 3.4	7.0
51- 75	52	+ 5.3	8.0
76-100	70	+ 2.6	8.4
101-125	26	+ 1.8	9.1
126-150	18	+ 6.3	7.0
151-175	7	+ 9.5	11.9
176-200	3	− 3.6	6.5
201-	2	+ 0.6	0.8
		TYPE A	
0- 25	99	+ 1.2	10.8
26- 50	74	+ 2.6	10.9
51- 75	50	+ 3.8	11.2
76-100	42	+ 1.0	10.9
101-125	42	+ 2.9	10.1
126-150	37	+ 0.5	11.0
151-175	25	+ 0.5	12.5
176-200	20	+ 2.0	8.6
201-	7	− 1.0	7.1

velocity of rotation and velocity of translation, the latter being uncorrected for solar motion and galactic rotation. This is shown by the uniformity of the values in the last column of the table. There is probably a slight tendency for the mean velocities taken with regard to sign to be slightly larger when the velocities of rotation are small. This effect is complicated by solar motion and galactic rotation. Since the solar apex lies near $l = 30°$ and since we have already found that, at least among the A's, large rotations predominate between $l = 320°$ and $l = 80°$, it is obvious that large rotations will occur preferentially in stars of negative radial velocity. There is no explanation for this and the physical reality of the effect is uncertain. In order to test this the mean residual radial velocities, obtained by applying a solar motion of 20 km/sec toward $\alpha = 18^h\,0^m$ and $\delta = +30°$, have been arranged for groups of equal rotational velocity. There is now no systematic progression in the values. (TABLE 1 B.)

7. The lack of correlation between $v_0 \sin i$ and galactic latitude suggests that the axes of rotation are distributed at random. Hence, the question arises whether we can assume a uniform rotation for all stars of a given spectral type and attribute the observed variation of $v_0 \sin i$ to the random distribution of the axes. The probability that the inclination lies between i and $i + di$ is proportional to the area of the corresponding spherical zone, or $\sin i\, di$. The probability that the observed velocity is between $v_0 \sin i$ and $v_0 \sin i + d\,(v_0 \sin i)$ is

$$W(v)\, dv = c \sin i\, di$$

But

$$v = v_0 \sin i$$
$$dv = v_0 \cos i\, di$$

Hence

$$W(v)\, dv = c \operatorname{tg} i\, (dv/v_0) = c[v/\sqrt{(v_0^2 - v^2)}]\, (dv/v_0). \qquad (6)$$

The corresponding theoretical distribution has a pronounced maximum at v_0 and is quite different from the observed. We can, however, analyze the observed curve by means of functions of the type of (6) and attribute to each a weight such as best to represent the observations. The weights will then be proportional to the percentages of stars having the assigned true rotational velocities v_0. The results, taken partly from the work of Miss Westgate, are given in TABLE 2.

TABLE 2

v	0-50	50-100	100-150	150-200	200-250	250-300
Percent B and O	27	53	15	4	1	0
Percent A	16	28	24	24	11	1
Percent F0-F2	30	50	15	4	1	0
Percent F5-F8	70	30	0	0	0	0

The distribution for O and B and early F are very similar. The A's seem to have a greater percentage of fast rotations, but this result rests only upon Miss Westgate's series of observations. The startling decline at F5 is unquestionably real. In the later types spectroscopic binaries, like W UMa, have been excluded.

8. There is a very remarkable correlation between the rotational velocity as determined in the usual manner from the absorption lines of MgII, HeI, etc., and the total widths of the bright H lines in Be stars. This correlation is demonstrated in TABLE 3 taken from one of my earlier papers.[32]

TABLE 3

Emission Lines	No. of Stars	Estimated Amount of Rotation
Double, average, and single, very broad	15	8.5
Double, very broad	7	7.3
Double, narrow, and single, average	6	5.3
Single, very narrow	6	3.7

Here maximum rotation, designated as 10, corresponds to about 250 km/sec.

TABLE 4 gives all stars for which R. H. Curtiss had measured the widths of the bright lines:

TABLE 4

Star	Rotation	Width of Hβ
f^1 Cyg	9	10.5A
π Agr	10	7.5
ϕ Per	10	7.0
1 Hev. Mon	7	6.5
β Mon	7	6.5
b^2 Cygni	9	6.0
ψ Persei	9	6.0
κ Draconis	8	5.0
γ Cass	10	5.0
c Persei	5	4.0
β Piscium	1	2.5
11 Camelop	2	1.0

These results leave no doubt that the correlation is very conspicuous. It must not be confused with the tendency of B-type supergiants to show narrow emission lines, often of the P Cygni-type.

There has been some discussion whether the widths of the bright lines are determined by rotation. The absence of violet absorption borders strongly militates against expansion as a major effect. But the frequent variations in V/R suggest, according to McLaughlin and Gerasimovič, that expansion and contraction are not negligible. The widths, as measured by Curtiss, correspond to velocities from 30 km/sec for 11 Camelopardalis to 320 km/sec for f^1 Cygni. These values are, on the whole, not very different from the ones which are found from the absorption lines. But in the best-observed cases, for example that of ϕ Persei, the absorption lines give roughly twice the rotational velo-

[32] *Ap. J.*, **73**, 98, 1931.

city of Hβ. This is consistent with the theory, developed later on, according to which $v \propto r^{-1}$. The radii of the shells are, according to all methods of study, of the order of twice the radius of the photosphere.

9. Among the Be stars are some which show sharp and deep absorption lines of H and some lines of other elements between the bright components or even when the latter are not visible. These objects have been designated recently under the term "shells" or "extended atmospheres." The radial velocities of these shells are, at least roughly, similar to the velocities of the stars as determined from the broad lines of He and from the Stark effect wings of the Balmer lines. Most of the shell spectra are variable in intensity and radial velocity. Some have appeared suddenly, as in Pleione and γ Cassiopeiae, others undergo periodic changes, as in ϕ Persei and perhaps HD 45910, while still others change slowly in the intensities of the "α Cygni" lines, as ζ Tauri, ϵ Capricorni, etc.

Relatively few of these stars have been investigated in detail. Almost all have very large rotations as determined from the absorption lines of HeI, and all have very *broad* emission lines, although not all have *strong* emission lines.

It appears reasonable to suppose that the gases which give rise to Be-type emission are predominantly present in rapidly rotating stars and that the correlation discussed in section 8 is due to inclination. The frequencies of TABLE 3 suggest that broad emission lines are much more frequent than narrow ones, and the spread in v_0 as observed for ordinary B stars is much reduced. Probably all normal main-sequence Be stars rotate rapidly. If this is so, then the absence of shell spectra among narrow-lined Be stars suggests that the gases are confined to the equatorial plane. However, the number of stationary shell stars is small, and I cannot verify the suggestion. It is certainly true that the most conspicuous shells are at the same time the most conspicuous rotational stars. For ϕ Persei Morgan's small-dispersion spectrograms show $v_0 \sin i \sim 400$ or 500 km/sec and γ Cassiopeiae, as well as Pleione, were always known as exceptionally broad-lined stars.

The physical properties of the shells have been described elsewhere.[33] Briefly, they are tenuous masses of gas whose spectrum shows marked effect of dilution and whose radii are of the order of two or three times the radius of the photosphere. The pressures are of the order of 0.1 bar, which is perhaps 10^3 times less than the pressure of the reversing layers. There is usually considerable turbulence, and the rotational velocities are always very much smaller than in the reversing layer. In several shells there is a pronounced effect of stratification: thus, in ζ Tauri the lines of SiII and MgII originate low in the shell and show appreciable rotation while FeII originates higher, where

[33] Struve, *Ap. J.*, **95**, 134, 1942.

conservation of angular momentum causes the rotational velocity to be smaller. Finally, He I 3965 and H occur at the highest levels, where the rotation is least.

This effect of stratification is also observed in the emission lines of several Be stars. The bright line of Mg II, always weak because of dilution, is usually much broader (when it can be seen at all, as in χ Oph and γ Cas before the recent outburst) than the bright lines of H and of Fe II. These differences in width can be logically attributed to conservation of angular momentum in a stratified shell. The stratification results from the fact that Mg II is suppressed by dilution at the higher elevations, while temperature-excitation and ionization suppress it in the reversing layer of an early B star, such as ζ Tauri or ϕ Persei. On the other hand, Fe II is favored by dilution. How H will behave is not known. However, it is likely that the pressures in the shells are sufficient to effectively wipe out the metastability of the 2s level, so that dilution should produce a weakening of the Balmer lines. The fact that the Balmer lines are strong in spite of this effect in such stars as ϕ Persei and ζ Tauri must be a consequence of the high abundance of H and the fact that turbulence will produce an extended flat portion in the curve of growth.

In a few shells there are definite indications of departures from equilibrium conditions produced by the depletion of the continuous radiation of the exciting star on the short-wave side of λ 911. This is true in the case of 14 Comae Berenices where the absorption lines of H are so affected and of HD 190073, where the emission lines are weakened.[34]

It is of interest to note that Be stars are unusually frequent in several clusters. In the Pleiades four out of thirteen stars of types B8 and earlier have bright lines. The ratio is one to three. For the normal B8 stars of the galaxy the ratio is about 1:123 and for B0 to B5 stars it is 1:15. A similar tendency has been observed by Trumpler[35] in the double cluster of Perseus. We shall have occasion to recall this feature later.

The rotational effect in spectral lines provides us with a miniature spectroheliograph for analyzing the surfaces of the stars. Each point on the contour of a rotationally broadened line corresponds in effect to a narrow strip on the projected stellar disk, parallel to the axis of rotation. If the distribution of light on the surface of the star is not uniform in longitude the contour of the line will not have its normal shape and will vary with the period of the rotation. Unfortunately, we cannot by such means investigate differences in light with stellar latitude—a problem which we shall touch upon later. But there is one phenomenon which is indeed susceptible to treatment by this spectroheliographic procedure. In a close spectroscopic binary the reflection effect causes

[34] O. Struve, *Pub. A.S.P.*, **54**, 11, 1942.
[35] *Pub. A.S.P.*, **38**, 350, 1926.

an unsymmetrical distribution of light on the surface of each component. As the components rotate the asymmetry shifts from one side to the other and, as McCarthy has shown, the contours of the lines should exhibit a corresponding variation. I believe that in α Virginis the effect is actually present: the weaker component of He I 4472 is clearly unsymmetrical at the nodes.

(To be continued)

Easter Intervals

By GEORGE W. WALKER

(Continued from page 179)

Up to now, we have been interested primarily in the present period, which, as we have said, comes to an end with the year 2199. Our calendar rules provide for something new to happen at that point, and at many other similar century points later on. With the help of Figure 1, the thing that happens can be stated very simply. In turning from the year 2199 to the year 2200, the whole circle of golden numbers, the inner circle of Figure 1, is turned, all together, one unit of angular advance, clockwise; thus bringing each of the golden numbers into correspondence with a new epact and a new full moon day.

At the end of another century, between 2299 and 2300, a shift of the same kind is made, again. Then, a century later, comes a shift of one unit in reverse. So, often, between centuries, the correspondences shown in Figure 1 will change, in an orderly fashion, as the calendar goes on.

The purpose of this slipping of the set of golden numbers is to keep the calendar in step with the moon through long periods. Its over-all pace is intended to compensate both for the occasional century leap years and for the slow drifting of the 19-year lunar cycle. So slow is the progress that it takes about 70 centuries to get all the way around the circuit once. Within the lengths of time that we are interested in for Easter intervals the golden numbers never get very far from their starting-points.

To allow for this new feature in the calendar, we may suppose that the inner circle of Figure 1, the circle of golden numbers, is also a separate disc, free to turn on its central pivot. Only, it seldom turns. It maintains whatever position it has for at least a century, and, often, for several centuries, at a time.

We have now noticed two separate things that happen at century points, which have direct bearings, and different effects, on our Easter intervals. One thing is the passing of a century common year, which drastically affects Sunday intervals, and has the effect of shifting the array of valid intervals downward in Table VII or Table XI to the

next following row of units. The other thing is the passing of a point of turning, which, primarily, brings a slightly different sector of the inner disc of Figure 1 into play. At some century points, one of these two things will happen, at other points, the other; at some century points, both of them will happen together, at others, neither.

We know, of course, by our leap-year rule, at what century points we encounter century common years, at three century points out of every four. For our Easter calculations we shall need to know also at what century points the turnings of the circle of golden numbers will occur.

We can see the sense in the calendar rules for this most clearly if we understand that there are two factors working together independently. A turning ahead is always indicated whenever a century common year arrives; and a reverse turning is prescribed every three or four centuries, eight times, spaced as evenly as possible, in every 2500 years.

The two causes, combining, result in a cycle of movements to and fro, which is repeated every 100 centuries. All the slipping-points, therefore, are geared to the century numbers (CN, for short), so that whenever a given century number comes around again, every 10,000 years, there will always be the same thing happening; either an advance, or a retreat, or a holding steady of the circle of golden numbers in Figure 1.

We can express what happens at any century point, or at any sucession of century points, symbolically, very simply. For example, we have already stated that when we start from a point in the present century (CN = 19), we hold Figure 1 steady for two centuries to come (until 2199), then we have an advance, another advance, and then a retreat. Symbolically, using the initial letters of the words only, we have: SSAAR.

If we go on into later centuries, beginning within any century whose CN is 19, we find the continuing pattern in symbols will look like this:

SSAARAASSASASSAARAASSAASS

and so forth. Similarly, we can write out the pattern for the series of century points beginning with any other century number.

Better yet, we can write out the graphical pattern for the entire sequence of 100 centuries. Then, to find the pattern for any particular century, we have merely to begin from the appropriate starting-point and carry on as far as we please. The entire series of slippings of the golden number circle forward and backward is graphically shown in Table XII. Pick out the sequence beginning from CN 19 and compare it with the symbolical pattern that appears in the paragraph above.

It is plain that when we deal with Easter intervals crossing century points it will make a difference in what century we begin. We have to watch for century common years, and for this new slipping of golden numbers besides. Really, we can save trouble if we classify all the 100

century numbers according to their patterns of behavior.

We start, naturally, by separating the century numbers into four classes according to their respective places in the leap-year cycle. This will take care of the century common years, at least. For this, we can use quartic remainders again, just as we did when we divided all years into four classes at the beginning of this research; only, this time, we are interested in the remainders left after dividing the century numbers, not the year numbers, by 4. We may call the quartic remainders of the century numbers QN for short.

TABLE XII*

SHOWING WHAT HAPPENS TO THE CIRCLE OF GN'S AT ALL CENTURY POINTS IN 10,000 YEARS

CN =	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20
	/	-	/	-	-	/	/	\	/	/	-	-	/	-	/	-	/	-	/	-	
	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
	-	/	/	\	/	/	-	-	/	-	/	-	-	/	/	\	/	/	-	-	
	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
	/	/	-	-	/	-	/	-	-	/	/	\	/	/	-	-	/	-	/	-	
	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
	-	/	/	\	/	/	/	\	/	/	-	-	/	-	/	-	-	/	/	\	
	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	00
	/	/	-	-	/	-	/	-	-	/	/	-	-	/	/	\	/	/	-	-	

*In this table the dash denotes "steady"; the line rising to the right, "advance"; the line falling to the right, "retreat."

Next, within each set of century numbers, we may arrange the 25 members in order, according to their respective rapidity in pushing ahead. The thing that matters is the total turning of the disc of golden numbers which can be accomplished in any given time. Those century numbers that begin with holding things steady at the home base, like

TABLE XIII

CENTURY NUMBERS ARRANGED IN ORDER OF SPEED IN ADVANCING THE CIRCLE OF GOLDEN NUMBERS

ON	QN3	QN2	QN1	QN0	ON	QN3	QN2	QN1	QN0
1	67	42	17	92	14	31	06	81	56
2	95	70	45	20	15	59	34	09	84
3	23	98	73	48	16	87	62	37	12
4	51	26	01	76	17	15	90	65	40
5	79	54	29	04	18	43	18	93	68
6	07	82	57	32	19	71	46	21	96
7	35	10	85	60	20	99	74	49	24
8	63	38	13	88	21	27	02	77	52
9	91	66	41	16	22	55	30	05	80
10	19	94	69	44	23	83	58	33	08
11	47	22	97	72	24	11	86	61	36
12	75	50	25	00	25	39	14	89	64
13	03	78	53	28					

the present CN, or even begin with a step to the rear, will affect our Easter intervals in one way; while those that lead to a rapid rushing ahead, like CN 21, or, to take the extreme case, like CN 64, will affect intervals in the opposite way.

Table XIII shows the ordinal arrangement of the CN's within each of the four classes; and the first column gives the ordinal number (ON, for short) for each CN. Each column begins with the most reluctant CN's, and lists last those that turn the disc farthest most quickly.

To be most useful, Table XIII should show, with each of the CN's in the four columns, the pattern of advances, retreats, and steady points, stemming therefrom. Any one who cares to can write out these patterns with the help of Table XII, as the pattern for CN 19 was written out above. One would find that he has to carry out the patterns of successive century points through 24 centuries in order to get the CN's all separated from their neighbors, so much alike are the patterns of several adjacent pairs.

We know now all about the locations and relative locations of the century points that turn or hold steady the inner disc of Figure 1. We should ask what this infrequent slow turning does to our Easter intervals. The answer can be stated quite simply.

Primarily, any turning of the disc of golden numbers, one way or the other, brings into the week preceding any given calendar day a slightly different sector of the inner circle of Figure 1, with, possibly, a different set of permitted GN's. If the disc advances, it brings up into the week one new earlier unit, thus enabling us to reach so much farther backward around the circle; while, at the same time, it cuts off the unit farthest ahead. If the pattern retreats, the effect is just the opposite.

When we apply this to the results that we have already worked out, we can take the whole thing into account by simply adding a new net angular advance to the entire list of net angular advances shown in Table IX, so that all the figures in the first column of Table IX are increased together by 1 (or 2 or more) or decreased by 1. To save confusion, we should call this new sort of angular advance, which operates on the whole circle at once, by another name. Let us call it the total turning (TT, for short). Like the AA, the TT is measured clockwise. So far as Table IX is concerned, the TT and the AA operate the same way, and may be combined. When they are added, we may call the combination TA for total advance.

We notice, when we combine any significant TT with all the AA's in Table IX, that we thereby displace somewhat the true center of the column, so that it is no longer found in the middle row shown in Table IX for which the golden number shift is 0.

Applying the results in Table IX to the next step, in which we made diagrams for the golden number shifts for Easter on a given calendar day, we find now that the whole row of cut-points (in both parts of Table X, for instance) moves one epact's distance to the left for every

added unit of total turning.

Saying the same thing in reference to Figure 2, we find that the point that we are now to take as the matching point moves one epact's distance clockwise around the margin of the inner circle for every unit of clockwise total turning. This has the effect of turning the disc shown in Figure 2 as many units in the opposite direction.

In making Figure 2, we innocently inserted some inconspicuous figures around the margin of the disc. With the help of these, the use of Figure 2 when a total turning is involved is just as simple as when there is none.

Having figured out from the century points the amount of total turning that we have to take into account, we look for the corresponding figure on the rim of the disc shown in Figure 2, and use that as the matching point for either end of the sector permitted for the full moon day. The final list of possible golden number shifts will include all that then come within the limits of the permitted sector in the two positions of the matching point. The valid cases (epact and golden number combinations) for each of the valid GS's will be those that are found on the farther side of the indicated cut-point, reckoning from the matching point as the home base.

Of course, for any particular GS, only that position of the disc has to be considered which will bring the cut-point for that GS within the permitted sector. If neither position of the disc will do it, there are no valid cases of Easter intervals having that GS.

As soon as we get involved in a total turning, we discover that the zero golden number shift has no longer any special position. This GS has become just one among several in a series, all conforming to the same rules. Whatever special character it had belongs now to the new center point in the column in Table IX, or to the new matching point on the disc of Figure 2.

Looking next at our latest table of valid Sunday intervals, in Table XI, we find here also a displacement of the center every time the interval absorbs a new turning point. For every added unit of total turning, the center is displaced to the left by a distance somewhat less than the distance between adjacent columns in Table XI.

Collecting all cases, we find that there is never a negative total turning (counterclockwise) of more than a single angular unit from any starting position. If the center of Table XI moves this single unit to the right, we find that our farthest possible reach in that direction, with the widest possible AA, can still be compassed within the right margin of Table XI as originally planned. We never reach a GS farther to the right than GS 2.

During the course of long intervals, the real center of Table XI may accumulate a considerable drift to the left. Then, to allow for all possible valid intervals, we may want to extend the left margin of Table XI an equal distance, beyond the column which has GS 17.

However, we may anticipate a bit, and report that as we work up to longer intervals we shall find the valid range constantly narrowing. As it finally works out, we shall never meet any valid uninterrupted Easter interval which does not come within one of the eleven columns originally projected for Table XI, namely, the columns for which the golden number shifts are, respectively, 17, 6, 14, 3, 11, 0, 8, 16, 5, 13, and 2.

In one way, the slow drifting of the central disc in Figure 1 is positively helpful for our Easter calculations. Because of this drifting, we can assume that all positions of the disc of golden numbers are possible; and, in fact, in the long run, all are equally possible. So we are not tied up to any peculiarity that may happen to be characteristic of the present arrangement of the GN's shown in Figure 1. We can approach the study of Easter intervals from a perfectly general basis. We can forget specific golden numbers and deal rather with epacts and the various positions of the disc shown in Figure 2.

When we want to deal with Easter intervals of various length, it should help us to know that all intervals having the same golden number shift, that is, all appearing in the same column in Table XI, will have exactly the same behavior in all matters that concern their horizontal placement. The differences among them will all be vertical differences, such as ceilings over or floors under their possible year endings, and things connected with their quartic numbers.

Now that we know all the habits and varieties of both vertical and horizontal limitations, we should be ready to apply our results to definite cases, to find whatever examples there may be of intervals of any assigned length for Easter on any given calendar day.

We have used the 35-year interval several times as an example. Let us put together all that can be said about the initial year of a 35-year interval, for a certain calendar day for Easter—let us say, for April 1. We can make six definite statements about such an initial year.

(1) For such a year, of course, April 1 must come on a Sunday.

(2) The quartic remainder of the initial year of any 35-year Easter interval must be 0, that is, the year number must be divisible by 4.

(3) The century point included in the 35-year interval must be a century common year; therefore, the quartic remainder of the century number (QN) must be 2 or 1 or 0.

(4) The year ending must be not less than 65, and not more than 99.

(5) The full moon day of the initial year must be included within certain definite days in the early part of the week preceding April 1, so that the angular advance associated with GS 16, combined with any total turning of the disc of golden numbers, will still be possible within the permitted week.

To these conditions, growing out of what has been said above, we should add another, namely:

(6) In order to have a certain year the initial year for an uninter-

rupted Easter interval of 35 years, there must be no shorter Easter interval possible beginning from that same initial year.

Now to apply these limiting conditions, to set definite limits.

Together, conditions (2) and (4) limit the possible year endings to just these eight: 68, 72, 76, 80, 84, 88, 92, and 96.

Condition (3) limits the possible century numbers to those given in the last three columns in Table XIII.

Condition (3), in conjunction with condition (1), puts another limitation on the possible year endings, also. In the 400-year cycle of the days of the week, only certain combinations of QN's and YE's will make April 1 Sunday.* If we study the various possibilities, we find Sunday on April 1 for only one or two of the eight listed year endings, for each QN. If QN is 2, YE must be 88; if QN is 1, YE must be 92; if QN is 0, YE must be either 68 or 96. And this last sentence sums up in a few words all that can be said or needs to be said about the vertical limits on the 35-year interval for Easter on April 1.

When we begin to work on the horizontal limits, with condition (5), we have several different factors to take into account. And, first of all, we should look into the matter of possible turnings of the disc of golden numbers.

Each valid 35-year interval must include one century point, and can not, of course, include more than one. The kind of century point included will be whatever kind immediately follows the century number of the initial year.

Taking, in turn, each of the CN's listed in the last three columns of Table XIII, we can look up the succession of events shown in Table XII, and, in particular, see what happens, with each of these CN's, at the century point immediately following. The result of the survey is interesting. If we draw a horizontal line through the last three columns of Table XIII between ON 8 and ON 9, every CN above that line will be followed by a century point that holds the disc of golden numbers steady; while every CN below that line will be followed by a century point that turns the disc one unit of angular distance ahead.

Therefore, to set the disc shown in Figure 2 to show the valid cases for 35-year intervals for Easter on April 1, we first use the point marked 0 as the matching point, for all century numbers that have ordinal numbers between 1 and 8; then we proceed similarly for all CN's below the line, with ON greater than 8, using the point marked 1 in the margin for the matching point.

We can sum up what we find as follows. When ON is not more than 8 (this case is pictured in Table X (A) and in Figure 2), golden number shift 16 (for interval 35) is valid for golden numbers 4 to 19, inclusive, with epact 16, and for all golden numbers with epacts 17, 18,

*For a definite procedure for locating the Sundays in a year with a given QN and a given YE, see *op. cit.*, "Easter Reckoning Made Easy."

and 19; and when ON is more than 8, GS 16 is valid for GN's 4 to 19, inclusive, with epact 17, and for all the GN's with epacts 18 and 19. This puts as concisely as possible all that can be said, or needs to be said, about the horizontal limits for 35-year intervals for Easter on April 1.

Every year that comes within the vertical and horizontal limits worked out above must have its Easter day on April 1, and will be followed just 35 years later by another year with Easter on April 1. We have the limits for all the valid cases, but we still haven't located or named a single year that will satisfy our specifications.

Let us pick out arbitrarily a definite set of specifications from the many possibilities offered and see if we can track down a year that will comply.

We can choose any century number from the 75 listed in the last three columns of Table XIII, and with each one we have one or two possibilities for the year ending, depending on the QN. Let us choose the CN at the top of the last column, that is, we choose CN 92, with QN 0 and ON 1. With QN 0 we have a choice of year endings. Let us choose YE 68.

In the horizontal range, we can take any epact-golden number combination on the counterclockwise side of the cut-point for GS 16; and when ON is 1 (or anything else up to 8) the cut-point is between GN 3 and GN 4 in epact 16. Let us choose the case nearest the cut-point, that is GN 4 and epact 16.

The vertical conditions alone, giving us CN and YE, tell us the last four digits of the year number we are looking for, namely 9268. Therefore the whole year number must be 9268 plus some multiple of 10000 years.

We might hope for a lucky break, and see if 9268 itself, that is, 9268 plus 0 times 10000 years, qualifies for epact and GN.

It is easy to find the golden number of 9268. Adding 1 to the remainder left after dividing 9268 by 19 gives us GN 16. There is no simple way of finding the epact for a specified year, but there are definitive rules.* We learn that the epact of the year 9268 is 13. We did not get the epact and golden number we hoped for, but we did gain a definite starting-point from which to reckon.

What does the addition of successive cycles of 10000 years each do to the epact and the golden number?

The golden number shift (GS) for 10000 years is 6. If we consult Table IX, we find that within any period not complicated by any total turning, GS 6 corresponds to an angular advance of —6 or —7, depending on the GN; in either case, an advance clockwise of 450 "degrees." (Compare Table XI, extended.) Hold this a moment.

Such a long interval as 10000 years is bound to have a considerable

*For a definite procedure, see *op. cit.*, "Easter Reckoning Made Easy."

accumulation of total turning, too. In fact, it has all the total turning in the entire table of 100 centuries shown in Table XII. We find that this TT, all told, amounts to 43 angular units clockwise. It is natural to drop out the completed circle of 30 units, and say that the net total turning is 13. This is the same as 247 "degrees."

Now we recall that the AA and the TT are to be added together. 450 + 247 — 570 = 127 "degrees." So the total advance from a cycle of 10000 years is always a total advance of 127 "degrees."

If we go forward another 10000 years, we have a TA of twice 127, or 254 "degrees." Three more added cycles carry us more than all the way around the circle. To find the net advance, we drop out 570 "degrees" whenever we can.

127 and 570 are incommensurable. Therefore successive additions of 10000-year cycles will continually bring new net total advances, until we have added 570 cycles in all, after which we are bound to repeat. The over-all grand cycle of Easters and of Easter intervals is 570 times 100 centuries, or 5,700,000 years.

It takes 193 added 10000-year cycles to bring about a net total advance of a single "degree." Twice 193 cycles give an advance of 2 "degrees." And so on. And in this inverse operation, as in the direct, we drop out 570 cycles whenever we can to get the net number of cycles needed.

We have data enough now to find a definite initial year of a 35-year interval for Easter on April 1. We know that the epact of 9268 A.D. is 13, and the golden number is 16. We want to add enough 10000-year cycles to move it to epact 16 and GN 4. Measuring off, we find that we want to bring about an advance of 525 "degrees" clockwise. So we multiply 193 by 525, drop out all the 570's that we can spare, and we find the answer, that we should add 435 cycles to the base year 9268.

Therefore, the year we were seeking, with QN 0 and ON 1, that is, with CN 92; and with YE 68; and with epact 16, and GN 4; is the year 4359268 A.D. A check-up shows that, in fact, both the year 4359268 and the year 35 years later have Easters on April 1.

If we want another initial year with the same specifications, we may add a grand cycle of 5,700,000 years and get 10059268; or two grand cycles and get 15759268; or as many grand cycles as we please.

We can take any other set of specifications, CN, YE, GN, and epact, within the vertical and horizontal limits, and, following the same procedure, we can find another year number. If we want all the cases which have the last four digits 9268, we can let the horizontal location range between the cut-point and the farther end of the permitted week, including all cases from GN 4 in epact 16 to GN 19 in epact 19. This means that we have all angular magnitudes for total advance from 525 "degrees" down to 453 "degrees." For these 73 TA's, we can easily find the 73 corresponding multiples of 10000 years, which, naturally, form

a pattern easily followed among the 570 multiples possible.

If we want a list of all the years that are initial years of 35-year Easter intervals, we may follow this same procedure for every permitted combination of CN and YE. The total number of 35-year intervals for April 1 within the first grand cycle of 5,700,000 years can be summarized by counting up the choices of particular specifications. These are counted in Table XIV.

TABLE XIV

SPECIFICATIONS OF ALL 35-YEAR INTERVALS FOR EASTER ON APRIL 1

QN	ON	YE	Ep.GN to Ep.GN	:	CN's	"Degrees"	Cases
2	1 to 8	88	16: 4 to 19:19	:	8	73	584
1		92			8	73	584
0		68			8	73	584
0		96			8	73	584
2	9 to 25	88	17: 4 to 19:19	:	17	54	918
1		92			17	54	918
0		68			17	54	918
0		96			17	54	918
							6008

Finally, we remember that we still have to take condition (6) into account. This bids us reject all cases in which the initial year is followed by a return of Easter earlier than the return that closes the interval we are interested in.

As it happens, none of the possible intervals smaller than 35 encroach on the area used by the 35-year interval; so all the 6008 cases just listed are uninterrupted. However, if we had been counting 40-year intervals, we should have found a number of cases that seem to meet all the specifications, but come to a premature end, some at the end of 35 years, others after only 5 years. Condition (6) requires us to weed these out.

The author knows of no way of taking condition (6) into account except to follow the procedure used in the sieve of Eratosthenes, that is, to work on smaller intervals first, and eliminate all cases that satisfy them, then figure out the limits for larger intervals only within the area that is left. This is what we meant when we spoke of narrowing the range.

A chart of the permitted vertical and horizontal limits for any particular interval-length for any given calendar day can be worked out, as in Table XIV, either on the basis of including all Easter returns, or on the basis of including only uninterrupted intervals. The author has a chart (rather voluminous, unpublished) giving the definite specifications for all possible uninterrupted intervals for all 35 Easter days. From this chart it becomes possible to furnish quickly concrete examples of any valid interval-length for Easter on any possible calendar day.

In the process of eliminating cases in which the interval is interrupted, we find that with some interval-lengths, every last case that we might expect to be valid is thus ruled out. For example, in the introduction we mentioned a return of Easter to April 1 after 22 years from

a preceding April 1 Easter (1934 and 1956). Of course, this is interrupted in the middle by this year's Easter which is also on April 1. We find that every single case of a 22-year interval in the entire grand cycle of 5,700,000 years is likewise interrupted in the middle. Therefore, although 22 years is valid for a Sunday return, and even provides numerous cases of Easter returns, we find no mention of a 22-year interval in our final list.

Among the valid intervals, we find other points of interest. Naturally, we would expect only a few cases of Easter intervals from the extreme columns of Table XI, with GS 2 or 17. As it turns out, there is just one valid interval-length with GS 2, namely, 40 years; and only 160 cases of these in 5,700,000 years. All of them are for Easter on April 19, which is the most favorable day for marginal cases. The earliest example of an uninterrupted 40-year interval has the initial year 66099 A.D. There is another which starts within 2500 years of that year, in 68589 A.D.

17-year intervals (with GS 17) are also quite rare. There are 373 cases for Easter on April 19, and 34 each for 9 different 7-epact Easter days, a total of 679 in all. The earliest occurrence of the 17-year interval is surprisingly soon, beginning in the year 5891 A.D. on Sunday, April 19.

The 40-year interval is the only one with GS 2. The 51-year interval is the only one with GS 13. There are 4 different interval-lengths, including three of the five longest, which have GS 17.

The shortest Easter interval is 5 years, which is also the shortest interval for a Sunday return. 5- and 6-year intervals are moderately common. Repete Easter is enjoying both currently, as was mentioned in the introduction.

Overwhelmingly the most numerous Easter interval is the 11-year interval, so common that if you should pick out a year at random, the chances are almost even that the Easter interval beginning from that year will be just 11 years. The interval in either direction from the current year, 1945, is 11 years, as we noted in the introduction.

In the process of narrowing the range, all the columns to the right of the zero column in Table XI are eliminated quickly. The only interval longer than two centuries in any of these columns is 293, with GS 8. The zero column only survives as far as interval 304.

The longest interval for Easter on April 19 is 79 years. The longest for any day between March 25 and April 23 is 152 years. Several of these days have no intervals longer than 119 years. For half of them 147 is the longest.

No interval for any day between March 25 and April 23 crosses more than a single century point; therefore there must be at least one Easter in every century for every one of these calendar days. Each of the seven other calendar days for Easter occasionally skips an entire century.

In the introduction, an Easter interval of 451 years was mentioned, beginning March 24, 1940. An even longer interval overlaps this, a 467-year interval for Easter on March 22, beginning in 1818. This record length will be equaled by another 467-year interval for March 22 beginning in 2505. After that, new records are established as given in Table XV.

TABLE XV

NEW RECORDS FOR EASTER INTERVALS (OF LENGTHS EXCEEDING 1000 YEARS)

STARTING-POINT		INTERVAL
March 23,	3124 A.D.	1059 years
April 25,	5120 A.D.	1363 years
March 22,	16212 A.D.	1431 years
April 25,	118928 A.D.	1499 years
March 22,	171812 A.D.	1887 years

The last entry in Table XV reaches the all-time high. However, if the days of the week had just happened to be tied to the calendar in a slightly different way, we might have had as many as 15 intervals of 1955 years in every grand cycle, or even 30 of these and 36 2023-year intervals besides. We might gain this maximum even yet if we can get universal consent to calling a certain Saturday Sunday, and going on from there, making no change otherwise in our calendar rules. Or, if this seems difficult, we could gain the same end much more simply, by agreeing to reckon the spring equinox for calendar purposes on March 20 instead of March 21. This last would seem quite a reasonable change, in view of the fact that the sun often "crosses the line" on March 20, as things go today.

But perhaps 1887-year intervals are long enough. Threepete Easter thinks 451 years is too long.

As the days of the week are geared to the calendar now, there will be 24 1887-year intervals in a grand cycle, and none longer. The author has a list of all 24. The earliest occurrence is given in Table XV.

There are two rivals for the claim of being the rarest of all possible Easter intervals, namely, 671 years and 1803 years, either of which has just 6 occurrences in 5,700,000 years. The initial years for the 1803-year intervals are 427412, 1783992, 2527412, 3713992, 4367412, and 5553992 A.D., and for 671-year intervals, 888920, 1596120, 2728920, 3526120, 4658920, and 5366120 A.D.

In any case when we have a year number given, we can write down by inspection additional instances by adding to the year number any number of full cycles of 5,700,000 years.

To finish this report, it only remains to add a complete census of all possible uninterrupted Easter intervals. The particular specifications would take too much space, but we might find room for the totals. This information is given in Table XVI.

In this concluding table, the 35 calendar days on which Easter might fall are arranged in the order of frequency. The number of epacts and

TABLE

COMPLETE COUNT OF ALL UNINTERRUPTED

Yrs. of Int.	Total No. of Cases	1 : 19° Mar 22 D	1+ : 30° Apr 25 C	2 : 38° Mar 23 E	3 epacts : 57° Mar 24 F	3 epacts : 57° Apr 24 E	4 epacts : 76° Apr 23 A	4 epacts : 76° Mar 25 G
5	376231						40	
6	262699							
11	2651307			7736	26173	26173	44546	44698
17	679							
35	137613						384	512
40	160							
46	321594			576	3168	3520	6675	7246
51	105478							
57	483462	2736	7686	10134	13500	12796	13884	12358
62	339255						160	160
63	21879							
68	234571	4080	9044	8543	7810	9160	7287	11060
73	316690				1024	1024	9240	10048
79	189786						1744	2416
84	76534	768	1824	6024	13951	14626	10376	12052
95	18132	4180	7328	5208	708	708		
119	1541							
125	495	495						
130	13316						384	512
141	46637			576	3552	3680	4371	4211
147	17297						170	
152	51087	4664	8967	9642	7783	7108	5569	3840
163	8409	1224	2907	1819	523	1046	238	652
179	456	64	304	88				
209	251					200		51
220	7728	2518	1506	1330	1330	1330	786	258
231	1015				432	288	241	54
247	792	792						
277	704	704						
288	403	16	72	72	72	144	27	
293	64						64	
299	26				10	10	6	
304	2008		528	616	360	368	88	48
315	88	88						
372	4991	3198	575	663	375	108	72	
383	519		8	24	198	265		24
451	133				85		48	
467	440	440						
524	700	308	154	154	42	42		
535	331		152	152	27			
592	506	240	112	112	42			
603	32			20	6	6		
671	6				6			
676	57	36		21				
687	337	64		201	24	48		
755	149			125	24			
896	111	48	42	21				
907	135	40	48	47				
975	62		40	22				
991	361	361						
1059	740	150	402	188				
1127	58		40	18				
1279	174	48	123	3				
1363	204	114	75	15				
1431	123	114	9					
1499	27		27					
1583	30	30						
1651	27		27					
1803	6	6						
1887	24	24						
	5700000	27550	42000	54150	81225	82650	106400	110200

XVI

EASTER INTERVALS IN 5,700,000 YEARS

5 epacts : 95°		6 epacts : 114°		Seven epacts or 133°			7+ : 141°	8 : 152°	Yrs.
Mar 26	Apr 22	Apr. 21	Mar 27	6 days	7 days	9 days	Apr 18	Apr 19	of
A	G	F	B	A, C*	E, F*	B, D, G*	C	D	Int.
1777	1750	7800	8400	14127	13975	15050	16727	21700	5
		442	442	9423	10292	10396	15023	24646	6
63071	63223	81748	81748	91671	91927	91823	94271	97998	11
						34		373	17
1656	2208	3081	4108	4506	4506	6008	5106	7908	35
								160	40
8406	8604	9831	9554	11256	11256	10504	11856	11294	46
		208	208	3480	4061	4282	5766	11451	51
15156	13104	16581	14054	16329	15900	13106	15455	8790	57
2523	2688	10498	10346	12202	13448	13296	12304	13564	62
		1326	1547	612	612	1156		646	63
7762	12960	7124	10882	3319	4700	8486	3017	6654	68
15422	17138	11292	12244	9000	9750	10804	9000	10772	73
4071	5669	7644	8994	5571	6698	7769	4571	4444	79
5625	7500	1652	1800		48				84
									95
	40		277			136			119
									125
1156	1378	523	396	854	427		854		130
2741	1012	1477	300	2354	1177		2354		141
896		923		1496	748		1096		147
2738	476	300							152
									.
									.
									.

*The figures in the three columns starred are for single days, like the others. The days to which they apply are the following:

A, C: March 28, April 2, 4, 9, 11, and 16.
E, F: March 30 and 31, April 6, 7, 13, 14, and 20.
B, D, G: March 29, April 1, 3, 5, 8, 10, 12, 15, and 17.

133000	137750	162450	165300	186200	189525	192850	197400	220400

the number of "degrees" covered by each is given. The letter appearing with each calendar day is the "Dominical Letter" by which one can list all the Sundays of any year.

The 7-epact days for Easter are so numerous and so much alike that they are here gathered into three columns which are distinguished by their dominical letters. The foot-note supplies the particular calendar days.

It is left to the reader to notice the significant or interesting things in this final table, such as the preference shown by certain intervals for certain days, the general location of short and of long intervals, the maximum intervals for different days, and the way in which this maximum changes between columns, the overwhelming frequency of the 11-year interval, and the order of frequency of other common intervals, the rapid reduction in the number of cases as we go to the longer intervals, the freak distributions for certain intervals, 95, 125, 293, *et al.*, the two intervals that are found in every column, and so forth.

There is enough interesting material in this table alone to supply Peter Easter and his family with conversation for many Easters to come.

PERRY, N. Y.

The Planets in June, 1945

NOTE: Greenwich Civil Time is employed unless otherwise stated. To obtain Eastern War Time subtract 4 hours, Central War Time, 5 hours, etc. The phenomena are described as they are to be seen from latitude 45° N. The data are taken chiefly from the *American Ephemeris and Nautical Almanac*.

Sun. The sun will continue its northward course until June 22. On this date it comes to a solstice, the summer solstice in the northern hemisphere. After this it begins a southward motion, quite slowly at first, and ends the month just 14′ south of the farthest point north. It will move eastward from $4^h\,34^m$ to $6^h\,34^m$.

Moon. The phases of the moon will occur as follows:

	d	h
Last Quarter	June 3	13
New Moon	10	4
First Quarter	17	14
Full Moon	25	15

It will be nearest the earth for the month on June 7 and farthest from the earth on June 20.

Mercury. Mercury will move from a position about an hour west of the sun at the beginning of the month to a position about an hour east of the sun at the end of the month. It will be in conjunction with the sun, on the opposite side of the sun from the earth, on June 16. It will, therefore, not be visible during this period.

Venus. The angular distance between Venus and the sun will increase until June 24, when Venus comes to a point of greatest elongation west. Venus will, therefore, continue to be the brilliant morning star during this month. It will be in the constellation Aries, a short distance southwest of the Pleiades.

Mars. Because Mars will move eastward more slowly than the sun, it will rise earlier from morning to morning. At the end of the month it will rise nearly four hours before the sun. It will be quite near the planet Venus at the middle of the month.

Jupiter. Jupiter will be in quadrature with the sun, 90° east, on June 9. It will, therefore, continue to be favorably situated for evening observation during this month. It will be a little more than an hour east and slightly south of Regulus.

Saturn. Saturn will be visible near the western horizon at sunset at the beginning of the month. After the middle of the month it will be too near the sun to be seen. Its setting point on the horizon will be very nearly the same as that of the sun.

Uranus. Uranus will be in conjunction with the sun on June 4. Consequently it will be lost in the rays of the sun throughout the month.

Neptune. Neptune will be in good position for evening observations. It will be in quadrature, 90° east of the sun, on June 25. It will be very near the meridian at sunset throughout the month.

Asteroid Notes

By HUGH S. RICE

PALLAS, VESTA, and CERES are observable during this period and all are well placed for observers in the northern hemisphere.

PALLAS is now going forward and is, in general, northeast of β Leonis, at an average magnitude of about 8.2.

VESTA is, in general, north and east of δ Virginis, at an average magnitude of about 7.8, and headed southeast. On the night of June 11-12 it is in conjunction with δ and is north of the star. It is closer yet (about 0°.2) in the early evening of June 13 for our time zones, and in the early morning of June 14 it is directly east of the star. While the observer has his telescope directed to this part of the heavens, he will find many nebulae and other objects a few degrees to the west and northwest of the asteroid—in Virgo and Coma. Many Messier numbers are located here. On the morning of June 20, VESTA passes very close (by about 0°.1) to N.G.C. 4900, located 1½° southeast of δ. N.G.C. 4900 is also known as H. I143 and is described by Dreyer as "considerably bright, considerably extended . . ." Thus minor-planet hunting can be combined profitably with the observing of star clusters and nebulae.

CERES is a few degrees east and south of ζ Virginis, at an average magnitude of about 8.1; in the middle of the period it turns and goes forward.

The following ephemerides are a continuation of the ones in the last issue, and are furnished by the Yale University Observatory.

ASTEROID EPHEMERIDES FOR 0^h U.T.

2 PALLAS

1945	α h m	δ ° ′	1945	α h m	δ ° ′
May 25	11 54.1	+20 3	June 14	12 8.6	+19 32
30	11 57.1	20 3	19	12 13.2	19 14
June 4	12 0.5	19 57	24	12 18.0	18 52
9	12 4.4	+19 46	29	12 23.2	+18 28

4 VESTA

1945	α h m	δ ° ′	1945	α h m	δ ° ′
May 25	12 48.1	+ 6 3	June 14	12 53.7	+ 3 43
30	12 48.5	5 34	19	12 56.7	2 59
June 4	12 49.6	5 1	24	13 0.3	2 13
9	12 51.3	+ 4 24	29	13 4.4	+ 1 24

1 CERES

1945	α h m	δ ° ′	1945	α h m	δ ° ′
May 25	13 49.6	— 0 4	June 14	13 44.6	— 1 35
30	13 47.5	0 21	19	13 44.8	2 6
June 4	13 45.9	0 43	24	13 45.7	2 39
9	13 45.0	— 1 8	29	13 47.0	— 3 14

Hayden Planetarium, American Museum of Nat. Hist., New York,
April 24, 1945.

Occultation Predictions for June, 1945

(Taken from the *American Ephemeris*)

The quantities in the columns *a* and *b* are given for the purpose of making these predictions useful for any place within 200 miles of the point indicated. The procedure is as follows: Subtract the longitude of the point given from the longitude of the place in question; multiply the result in degrees, taking the signs into account, by the quantity under *a* for the star to be observed; similarly, with the latitude, using *b*; apply the sum of the products, with its proper sign, to the Greenwich C.T., and obtain the predicted Greenwich Civil Time for the phenomenon at the place of observation. To obtain Eastern Standard Time it is necessary to subtract five hours; Central Standard Time, six hours, etc.

Date 1945		Star	Mag.	IMMERSION Green-wich C.T.	*a*	*b*	Angle E from N	EMERSION Green-wich C.T.	*a*	*b*	Angle E from N
OCCULTATIONS VISIBLE IN LONGITUDE +72° 30′, LATITUDE +42° 30′											
June	2	29 Aqar	6.5	7 37.1	—1.4	+1.5	64	8 55.4	—1.8	+0.8	256
	28	17 Capr	5.9	3 55.9	—1.2	+1.2	88	5 11.6	—1.7	+1.1	255
OCCULTATIONS VISIBLE IN LONGITUDE +91° 0′, LATITUDE +40° 0′											
June	1	27 Capr	6.2	8 53.3	—2.5	—3.7	153	9 12.6	..	..	180
	2	29 Aqar	6.5	7 11.3	—0.9	+1.6	70	8 22.4	—1.4	+1.3	259
	13	μ Canc	5.4	1 44.4	—0.9	—0.3	57	2 23.3	+0.6	—2.6	333
	24	116 B.Oph	6.3	7 7.9	—1.7	—0.5	74	8 21.2	—1.4	—1.9	302
OCCULTATIONS VISIBLE IN LONGITUDE +120° 0′, LATITUDE +36° 0′											
June	13	μ Canc	5.4	1 17.8	—1.3	—1.6	110	2 29.3	—0.8	—1.5	283
	23	73 B.Scor	6.4	2 56.9	—1.4	+1.0	92	4 8.8	—1.0	—0.9	321
	24	116 B.Ophi	6.3	6 2.7	—2.3	+0.2	97	7 32.6	—2.2	—1.0	297
	27	329 B.Sgtr	6.1	10 37.8	—1.0	+2.1	21	11 25.6	—3.0	—2.6	311
	27	336 B.Sgtr	6.5	12 4.9	—0.3	+1.6	22	12 50.4	—2.2	—2.7	306

OCCULTATIONS VISIBLE IN LONGITUDE +98° 0′, LATITUDE +30° 0′*

June	2	29 Aqar	*m*	6.5	6 53.4	−0.5	+0.8	96	7 57.8	−1.2	+1.8	238
	13	*μ* Canc		5.4	1 46.8	−0.6	−1.1	95	2 45.6	+0.1	−1.5	297
	19	566 B.Virg		6.5	23 15.1	−1.7	+1.0	92	0 25.2	−0.9	−2.1	337
	23	73 B.Scor		6.4	3 43.6	−3.7	+1.7	66	4 44.8	−1.1	−3.1	346
	24	116 B.Ophi		6.3	6 59.4	−2.4	−0.7	93	8 26.9	−1.7	−1.3	284
	27	329 B.Sgtr		6.1	11 0.6	−1.3	−0.2	70	12 12.2	−0.7	−0.5	256

*Computed by Edgar W. Woolard and Paul Herget; communicated by Commodore J. F. Hellweg, Superintendent U.S. Naval Observatory.

METEORS AND METEORITES

Meteor Notes from the American Meteor Society

By CHARLES P. OLIVIER, President

The last list of new members appeared in the March, 1944, Meteor Notes. Since then we have had 60 persons join the A.M.S., the following list being complete to date. This is a most healthy increase and it is to be hoped that most of these persons will make real efforts to become active members in fact as in name. Some have already done something, but, as usual, the majority have allowed other things so far to interfere too much. I had hoped to have the 1944 annual report ready but this proved impracticable. Also there is reason to think a few members have neglected to send in their reports.

LIST OF NEW MEMBERS

Lurie, Robert, 116 Fort Pleasant Ave., Springfield 8, Mass.
Graham, J. L., R. 3, Box 807C, Salem, Ore.
Smallwood, W. W., P.F.C. 15341440, U.S.A.
Osburn, R. J., 2335 Lincolnwood Drive, Evanston, Ill.
Mercier, F. J., 151 Fairlawn Ave., Waterbury 45, Conn.
Gilkison, Charles, 4424 Emerson Ave., Dallas 5, Texas.
Atwood, J. E., 5160 Gloria Ave., Eucino, Calif.
Madans, Allen, Hayden Planetarium, New York, N. Y.
Upjohn, Dr. L. N., The Upjohn Co., Kalamazoo 99, Mich.
Stockmann, Cliff, 335 E. Utica St., Buffalo 8, N. Y.
Weller, Lawrence, 5000 Broadway, New York 37, N. Y.
Strauss, Allen, Jr., 222 W. 83 St., New York 24, N. Y.
Stowe, Bruce, 17 Beechmont Ave., Bronxville 8, N. Y.
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McDowell, Roy, R. 3, Barnesville, Ohio.
Kampe, Mel, 1324 Temple Pl., St. Louis 12, Mo.
Mary, Donald, 751 Convention St., Baton Rouge, La.
Denson, A. C., P.O. Box 122, Rockville, Conn.
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Allison, L. W., Danby, Vt.
Crone, L. H. W., 4015 W. Franklin St., Baltimore 29, Md.
Calabria, Joseph, Paul Revere Boys Club, 339 Wayne St., Jersey City, N. J.
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Davis, Miss I. L., 67 Jane St., New York 14, N. Y.
Parmenter, B. C., 14014 25th St. N.E., Seattle, Wash.
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Today's mail brought in two letters of special interest. One contained a full news report from New Zealand concerning the results obtained by our associate member, R. A. McIntosh, on the bolide of 1944 December 30 which appeared over that country. The path was given and an account of its 20-minute, long-enduring train. He had over 400 persons send in data on this object. The other was on the daylight fireball(s) of 1944 August 1, 6:35 P.M., C.S.T., by Dr. W. Dieckvoss, formerly of the Hamburg Observatory, Bergedorf, but now a prisoner of war at Como, Mississippi. By a recent ruling of the War Department this latter could at last be reported to me. Our regional director for Florida and West Indies, J. Wesley Simpson, has also sent me most interesting sets of observations made by him when on trans-Atlantic flights. For these the plane's place cannot be given, but this does not affect the value of the hourly counts. Our colleague, J. Fraser Paterson, F.R.A.S., of Broken Hill, Australia, who incidentally sent in the clipping mentioned above, enclosed the following report on a short, unexpected meteor shower observed by him. It will be interesting to learn eventually if anyone else in the Southern Hemisphere had the good fortune to see it.

"A meteor shower was observed on Sunday evening, 11 February 1945, between 8:10 P.M., and 9:30 P.M. (local time). Fourteen meteors were counted—all of the 3rd, 4th, and 5th magnitudes. The outstanding features about the shower were: short trails, very rapid, and all of less than a second duration. Five of these meteors were little more than flashes of light. All of this group were either yellow or yellow-white, except one—white—which appeared at the zenith and travelled towards Carina. The radiant is somewhere near Epsilon Carina or Delta Vela. Had no time to get my charts out, so, had to be satisfied with a mere count. Fortunately the shower was also observed by my wife, and a visitor, E. J. Fotheringham of the R.A.A.F. Although we had the above area under observation until 11 P.M., no meteors were observed after 9:30 P.M."

We recently received a long report on his regular observations from the, at present, most active member of the A.M.S., namely, Prof. Mohd. A. R. Khan of Begumpet, India. His work is most valuable and should greatly encourage meteor observing among his countrymen.

These reports are typical of the wide diversity of what is received here on meteors and prove that valuable work is being done despite the unsettled conditions in world affairs. A discussion of certain meteors, seen on 1943 June 21 from New York or nearby, and all at about the same hour follows.

Meteors of 1943 June 21/22

Most of our publicity campaigns asking for data on a brilliant meteor pay well for our trouble, but occasionally the results are somewhat confusing. A typical example is furnished by those seen about 9:00 P.M., E.S.T., on this night. Having received an excellent report on a fireball seen at 9:11 P.M., by our member, J. N. Milnes, of Oneida, N. Y., I had notices put in the New York and New Haven papers asking for further reports. From Oneida it was twice the size of Venus and so should have been conspicuous over eastern New York. To make a long story short, I have recently finished a study of the 27 reports that arrived, and it proves that somewhere near 9:00 P.M., on that night there were several meteors bright enough to attract attention. These were:

(A) The fireball seen by Milnes and not reported by others.

(B) A fireball seen from New Haven by 3 observers, going, as was A, from S to N in the E.

(C) A fireball seen from Danbury, Hamden, and Waterbury, Conn., moving from N to S in the E (all good reports) and one poor report from New York City also.

(D) A bright meteor or fireball, to the SW from New York City, seen by 5 observers there and one in Montclair, N. J., none of the reports giving anything but general notes as to position or motion.

In addition we have 4 reports which are so incomplete that it is impossible to know to which, if any, of the above four fireballs they refer.

I began the study hoping that A and B were the same object. This is not possible as A has the excellent path: $\alpha_1 = 261°$, $\delta_1 = +13°$, $\alpha_2 = 275°$, $\delta_2 = +59°.5$. From these I calculate for $\theta = 226°.5$, $a_1 = 305°$, $h_1 = 47°.8$, $a_2 = 223°.5$, $h_2 = 56°.5$. From New Haven, by roughly combining the 3 reports, we obtain: $a_1 = 286° \pm$, $a_2 = 255° \pm$, $h_1 = 38° \pm$, $h_2 = 40° \pm$. As *both* fireballs seemed to rise, and were going in the same direction, we might *assume* they entered our atmosphere in parallel paths. But the coordinates for B are far too uncertain to justify turning them into right ascensions and declinations, and then securing a radiant from the two plotted paths. As for C, it is reported as going in the direction N to S, yet why observers saw B and did not see C is a mystery, as they were both in the eastern sky. Four persons, three of them giving detailed observations, could hardly, however, all have been wrong in which way it went! As for D, the reports are entirely useless for anything further than was given.

Had observers only recorded their times—if given at all it is "about 10" or "shortly after 10" (they used E.W.T.)—that datum would aid in decisions on identity. I think the only safe conclusion is that, within a space of perhaps 20 minutes, at least four meteors appeared, bright enough to attract attention, in the area mentioned. Further, probably A and B came from the same radiant.

Flower Observatory, Upper Darby, Pa., 1945 April 14.

Contributions of the Society for Research on Meteorites

Edited by FREDERICK C. LEONARD
Department of Astronomy, University of California, Los Angeles 24

A Legal Treatise on Meteorites; or, Did You Ever See a "Tomanowas"?

ROY R. CARPENTER
1903 Elm Street, Denver, Colorado

[*Continued from the April Issue*]

To substantiate the claim that this meteorite had become personalty of the Indians and then had been abandoned by them, the defendant, the Oregon Iron Company, had one, Susap, a Klickitat Indian, 70 years of age, appear as a witness for them; he testified in substance that he was about the last of his tribe and that, as a young boy, he used to go hunting with Wachino, a Klickitat chief; that he often saw this meteorite; that there were a lot of trees around it, measuring from 3 feet to 7 feet, as shown by the present stumps; that the old chief had told him and other Indians that the object in question was an idol; had a hole in it; then, when it rained, water fell into the hole, and the Indians washed their faces in this water and put their bows and arrows into the water when they went to war; that the medicine men said it had come from the Moon and that the Indians called it "Tomanowas." Another Indian, Sol Clark, 47 years of age, testified that his mother was of the Wasco tribe and that she had told him that there was a place up there where the Indians used to go to this "Tomanowas"; that they would send their young people up there in the dark; that the "Tomanowas" was a kind of bowl or rock with holes in it; that it was a kind of magic or medicine rock and belonged to the medicine men of the tribe; but the witness claimed no interest in it. So much for the testimony on this point.

The court stated further to the effect that the attorney for the appellant argued with much zeal that it was inferable that the meteorite was an Indian relic, that it was an object of worship, a—"Tomanowas," and thus the Indians must have dug it up sometime; that they must have moved it and erected it in a standing position where it was now found, and that they must have carved out the interior into those "fantastic pot-holes"; that they had venerated it, and used it in their warfare, and thus, and for such reason, and thereby, they had severed it from the soil and had appropriated it to their own use, rendered it personal property in their hands and then presumably had forsaken and discarded it, and thus it had become abandoned property and, as such, was the property of the finder, this appellant. But to such reasoning the court said, "What is there to show this court that the Indians ever dug it from the Earth, ever erected it, or that they ever carved out those holes? No witness said they did, and all that has been related concerning its use is traditional." Such evidence is too meager to prove a substantial fact, necessary to predicating the defendant's claim. Nature does many fantastic things, and presumably these very things are the result of natural causes, and the cavities contained therein are, after all, attributable to the same agency, to nature as against the presumption that the Indians did all this. Hardly can it be conceived that they hollowed out the pot-holes in its crown, in

view of the virtual impenetrability of the substance and the primitive tools and implements with which they had to do their work. So, if it be conceded that the meteorite was an object of Indian worship, the fact does not afford reasonable inference that it was severed from the soil by them and appropriated by them. True; they may have worshiped and utilized it, dipping their bows and arrows and laving their faces in the water accumulated in these bowls; but all this they could well have done without any appropriation; for tradition tells us that they worshiped Mount Hood and other immovable objects as they existed in a state of nature; and there certainly could not have been any severance or appropriation of Mount Hood and other immovable objects! We conclude in this case, then, that there was not sufficient evidence even to go to the jury, from which the jury would have been permitted to infer that this was once Indian property and that the Indians had later abandoned it or that it was an Indian relic lost by them. There being no error in the record, the judgment of the trial court is, therefore, affirmed.

The court, in aiding itself in this decision, seemed to be able to find no decision relative to meteorites other than the Iowa case, Goddard *v.* Winchell; and the law propounded in that case was the following, as was noted in the first syllabus of the report, which reads thus: "A meteorite or an aerolite, tho not buried in the Earth, is, nevertheless, real estate, belonging to the owner of the land, and not personal property, in the absence of proof of severance." Mere evidence of tradition that Indians revered this meteorite, washed their faces in it, and treated it as a magic or medicine rock, did not establish proof of severance.

I have made no search of the statutory laws of any State, but I cannot conceive that a legislature would have any reason to enact a law applicable to meteorites. I consider that the well-reasoned principles announced by the court in the cases of Goddard *v.* Winchell and the Oregon Iron Company *v.* Hughes would prevail everywhere in our land. To show how thoro the judge of the trial court and the supreme court were, in considering the Iowa case, it should be stated that they cited 3 unofficially reported cases, which I shall now review.

An unofficially reported case is found in the 2 issues of *The Albany Law Journal* for March 17, 1877, and July 28, 1877, Vol. 15, p. 216, and Vol. 16, p. 76, respectively, wherein the property right to a meteorite was litigated. The facts as given are as follows.

In 1875, a large meteorite, in passing over Iowa County, Iowa, dropped a fragment weighing about 75 pounds onto a public highway running across lands owned by the Amana Society, near the town of Homestead. Soon after, a man by the name of Henry Maas found the meteorite on the highway and placed it on exhibition in the store of that Society. When he went to get it, they held it as their property, claiming title under the law of accretion; that the stone had fallen onto land to which they held fee-title; that the fee in the highway was in the Society, subject only to the easement of public travel; that all accretions on the highway belonged to the realty as much as if made on the inclosures of their lands. Maas claimed the fragment by right of discovery, and that there was no prior owner, at least within the jurisdiction of a mundane court! After a long trial, the court decided that the meteorite was a part of the realty owned by the Society and belonged to the owners of the land.

After having thus acquired the viewpoint and the legal aspect of the meteorite in American courts, let us take "passage" on one into a French tribunal; in

doing so, we will not disturb the Irish wit of the editor of *The Irish Law Times* as presented in the issue of July 19, 1879, Vol. 13, p. 381, as follows:

"Property in Aerolites

"A knotty legal point had to be decided by [the] Tribunal of Issoudun (Indre, France) a few days back. On one of the very rare fine nights of this very wet summer, a peasant, crossing a field, saw what is commonly called a falling star; but it was one of unusual magnitude, made a great noise, and touched the Earth within a few yards of his feet. Frightened as he was, he went to the spot and dug up a stone of considerable size, in scientific language called an aerolite, which had buried itself 2 feet underground. The rural mind is now relatively instructed, and it occurred to this country man that what he had found, what in fact had dropped from heaven in his sight, must be a rarity and might have a money value. After consulting the schoolmaster of his commune, he took the mysterious substance of no terrestrial creation to the Issoudun Museum, and there received in exchange for it the, to him, wonderful sum of 250 francs in hard money! Short-lived was his joy! The proprietor of the field, who resided in Paris, brought an action. He claimed either the restitution of the aerolite, which fell upon his land, or 10,000 francs damages, which he judged to be the value of it. Mr. M. Charbonnel, an eminent Paris advocate, was retained for the clown who picked up the aerolite, and M. Bolle, an *avoué* of Issoudun, for the proprietor of the land. As no precedents in point were known to be in existence, the judges had to proceed by analogy, and the case was assimilated to that of alluvial soil brought by an inundation. If the advocates did not bring forward the example of the Deluge, one, at least, he who appeared for the landlord, went as far back as the *Institutes* of Justinian, quoting the chapter *'De Thesauris,'* Book IX: *'Nemo audeat in alieno agro opes abditas,' etc.:* 'Let no one steal the wealth buried in another man's field.' The tribunal could not, of course, resist such authority and such a display of learning, and sentenced the peasant to give up the 250 francs and pay the costs! This seems to us rather hard on the finder, for, if even the Roman law of treasure-trove was applied, according to the general rule, one-half should go to the finder and one-half to the owner of the land; but it seems to be in conformity with a recent American decision which was not mentioned. The case to which we refer, decided last year, was in regard to the property right in a meteor[ite] which dropped a fragment weighing about 75 pounds," *etc.* The remainder of the article is a recital of the case of Henry Maas *v.* Amana Society, which I have already given.

A correspondent in *The Albany Law Journal,* in the issue of October 11, 1879, Vol. 20, p. 299, writing on "Property in Aerolites," says in effect: "In a recent number, you notice a case which has arisen in France concerning the ownership of an aerolite discovered by a French peasant; that it is 'an entirely new question of property law.' The Roman Law XXX provides that in ordinary cases treasure (*thesaurus*) should belong one-half to the discoverer and one-half to the owner of the soil upon which it was found. There has always, however, been a dispute as to what was a treasure; a *'Thesaurus,'* says Paulus (*Digest.,* XLI, 1, 9), *'est vetus quaedam depositio pecuniae* [that is, 'of anything valuable'], *cujus non extat memoria; ut jam dominum non habeat.'*

"Now this recent aerolite is not the first descendant from the heavens which has been the subject of controversy in sublunary tribunals. Marcadé mentions a case translated as follows:

"'So was it, that the aerolite which caused so much excitement in September, 1842, which Pierre picked up upon the field of Jacques, belonged neither in whole nor in part to the proprietor of the field. A stone fallen from the heavens cannot be an accession of the land upon which it alights; it belongs entirely by occupation to him who has found it, as was decided by M. Duranton.' Marcadé adds in words which, when read by him, will be galling to M. Bolle, the eminent *avoué* of Issoudun, who has, you say, been retained by the land owner, [that he, Marcadé] 'can hardly conceive how an advocate could have been found to entertain the contrary opinion.'

"This, I take it, is one more illustration of the venerable saying, 'There is nothing new under the Sun.'

"Yours obediently,
"R. D. McGibbon,
"Montreal, September 24, 1879."

Evidently this correspondent had in mind the previously cited case in *The Irish Law Times*.

The foregoing represent every case in court that I have been able to find cited or in any manner referred to. Thus it will be seen that meteorites have been rare visitors indeed to the courts of law.

I am prompted to wonder whether a court in Jesus' time might not have come to much the same conclusion as our modern courts did in the Iowa and Oregon cases, for did not St. Matthew, in one of his parabolizing allegories as to the kingdom of heaven, indicate what may have been the prevailing rule of thought as to how to obtain riches in the land of another, when he said, "Again, the kingdom of heaven is like unto treasure hid in a field; the which when a man hath found, he hideth, and for joy thereof goeth and selleth all that he hath, and buyeth that field" (*Matthew* 13: 44)?

In conclusion, we may say that, until such time as a meteorite shall have been severed from the land of its fall or origin, if it be carried there by nature, it is realty. Of course, when the owner of the land severs a meteorite therefrom, as he would in the case of quarrying stone therefrom, or felling a tree thereon, and disposes of it, he makes it thereby personalty, and thereafter the law of personalty obtains.

Thus readeth the law from legal records, official and semi-official; and you have had the pleasure (?) of "digging" for meteorites where you have never dug before! And now I shall leave you to search in other, and still other fields, verily, verily with the birds of the air, the animals of the Earth, the fish of the sea, and to other ports of call, wherever these "heavenly beacons" may direct your ships of scientific search!

[*Concluded*]

The Train of the Richardton, North Dakota, Aerolite, June 30, 1918

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ABSTRACT

This paper contains a report of the writer's observations of the train of the Richardton, North Dakota, aerolite of June 30, 1918, a discussion of the observations, and the results derived therefrom.

The only references relating to the Richardton, North Dakota, aerolite which

I have been able to find are 2 articles by T. T. Quirke of the University of Minnesota. One, in *Economic Geology*, **14**, 619, 1919, deals with the recognition of metallic copper in this fall; the other, in *The Journal of Geology*, **27**, 431, 1919, describes a field investigation made by the author and gives data on the finding of various pieces of the meteorite, reports of various observers who witnessed the fall, and a determination of the path. Among the notes of witnesses, only 2 mention the train. Mr. Leo Kern reports "a trail of smoke or steam," and Mrs. Nell F. Rodenbour, of Lemmon, South Dakota, mentions the train in the following words: "A tail of light which remained for fully 15 minutes." As I happened to see the train and made some notes on it at the time, it seems desirable to call attention to them. The long delay before making this report is due to the apparent loss of the notes just after they were made and their recovery only a few months ago.

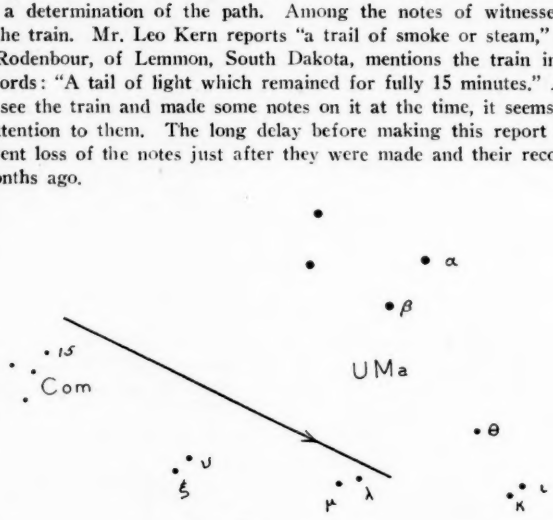

FIGURE 1

On the night of June 30, 1918, I was visiting at the home of Mr. Charles Delabarre, which is located in Sec. 14, Twp. 133, R. 90 W., North Dakota. The room was suddenly illuminated by a brilliant light which came from outside and which lasted for about 2 seconds. On hurrying outside, I saw a brilliant train that had appeared in the sky, as shown in Fig. 1. It appeared perfectly straight and had a diameter of approximately half a degree. It is difficult, after this lapse of time, to make a precise estimate of its brightness, except to say that it was considerably brighter than the brightest auroral streamer I have ever seen. I think that an exposure of one second with an *F*. 2 lens and a moderately fast modern film would have been ample to give a good photographic impression. The color of the train was about that of a G-type star. The meteor itself must have been brighter than magnitude —15.

An attempt to count seconds from the flash to a hoped-for sound resulted in a count of 240. This count was far from accurate, as it involved an attempt to allow for the time from the flash until I saw the train and to allow also for breaks in the count while I asked my friends, who followed me outside, to listen for the sound as well as to answer a question or two. Then the sound came. There was a strong general background of noise with occasional loud reports. The general background was of such intensity that it would have been necessary to shout directly into the ear of another if one had tried to speak intelligibly. There were possibly half a dozen of the extra-loud reports. After some 15 or 20 seconds, the sounds began to fade and ceased entirely after about half a minute.

Immediately after the sound had ceased, I asked for paper and made a diagram of the train among the stars. This diagram with the stars noted and later identified is given in Fig. 1. The notes on the diagram give the time of the fall as 11:45 P.M., Central Summer Time, June 30, 1918; the beginning of the sound as 11:49; and the ending as 11:49½.

The train was watched carefully while waiting for the sound. Little or no drift was noted up to the time the diagram was made, which I assume to have been 11:50, but later the train broke up into 6 sections which still kept their general alinement. The train was easily visible for 30 minutes.

On the assumption that the diagram was made at 11:50 P.M., Central Summer Time, and that my point of observation was in latitude N. 46° 20′, longitude W. 101° 55′, I find that the beginning of the train was at an altitude of 41°.2, azimuth 95°.4, and that the end of the train was at an altitude of 25°.6, azimuth 129°.8 (azimuths being measured from the the south point).

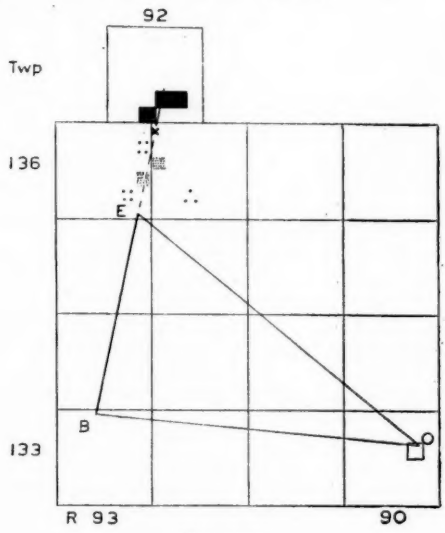

FIGURE 2

In Fig. 2, there are shown the sections (solid) where the larger masses, up to 18 pounds, fell, and the sections (dotted) where smaller fragments were found, as given in Professor Quirke's article in *The Journal of Geology*. He reports the center of the area of the fall to be in lat. N. 46° 37′.5, long. W. 102° 16′.3. This point was located as nearly as possible and is indicated by the small cross. From this point, which is assumed to be in the plane of the path, a line is drawn bearing S. 12° W., the reverse of the path determined by Quirke from the various observations. This line is assumed to be the projection of the path on the ground. From my point of observation, *O*, lines having the bearings of the beginning and end of the train, *OB* and *OE*, are drawn. The horizontal distances, *OB* and *OE*, measure 20 and 23 miles, respectively. From the observed altitudes, the heights of the train over *B* and *E* are 18 and 11 miles, respectively, and the length of the

train, $B'E'$, is 15 miles. These values are shown in Fig. 3. The angle which the train $B'E'$ makes with the horizontal is 28°, which agrees very well with the 27° found by Quirke from entirely different data.

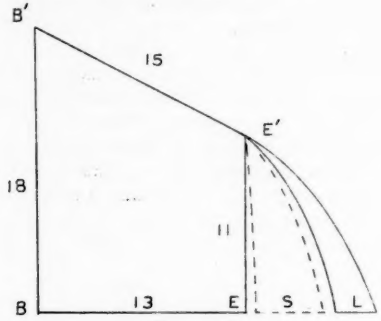

FIGURE 3

Various observers reported that the meteorite exploded and was no longer visible afterward, altho they could hear pieces flying thru the air. If we assume that disruption occurred at the end of the train, E' in Fig. 3, then the small specimens found fell almost vertically toward the line S, while the larger specimens reached the region of line L. Here S and L represent the projections of the areas in which the specimens were found on the extension of the line BE (Fig. 2) beyond E. In the case of the 3 pieces collected in the section farthest east, it seems that the disruption brought sufficient force into action to throw the pieces about 3 miles to the right of the original line of travel, in addition to nearly eliminating the forward motion. In the case of the 4 pieces found in the section farthest west, the disruption resulted in an almost vertical fall from a height of 11 miles.

The diameter of the train close to the end was noted as about half a degree. The air-line distance to this point from O (Fig. 2) was 25 miles, and the diameter of the train was therefore a little over 0.2 mile.

In C. P. Olivier's list of "Long-Enduring Meteor Trains" (*Proc. Am. Phil. Soc.*, **85**, No. 2, 93-135, 1942) he lists a total of 1336, dating from 32 B.C. to February, 1941 A.D., half of them having been recorded since the beginning of 1900, but the Richardton train is not mentioned. Of this number, there are 85, the heights of whose end-points are determined. Only 8 of the 85 have end-points as low as 20 km. (12.4 mi.). Since the end-point of the Richardton train was only 11 mi. (17.7 km.) high, it belongs to the small number of trains which extended down into the lower stratosphere.

March 6, 1945

Working Definitions of Fundamental Terms in Meteoritics

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The following working definitions of several of the most important terms in meteoritics have been in common use by many students of the subject since the foundation of the Society for Research on Meteorites in 1933 and carry, I believe,

the general endorsement of its Committee on Terminology. It is hardly necessary to add that all these terms are consistently employed in *C.S.R.M.* with the meanings given here.

Meteorite (adj., *meteoritic*): (1) a solid body of subplanetary dimensions existing as a discrete unit in space—called specifically a *meteorite in space;* or (2) a solid body from space that is falling or has fallen as a separate unit onto the Earth or onto some other astronomical body—called specifically a *falling* or a *fallen meteorite.*[1]

Meteor (adj., *meteoric*): the incandescence that results when a meteorite in space collides with the atmosphere of the Earth or with that of some other astronomical body and becomes, usually, a falling meteorite.

Meteoritics (adj., *meteoritical*): the science of meteorites[2] and meteors.

Meteoriticist: a specialist in meteoritics.

The single term *meteorite,* in the two preceding senses, evidently suffices to designate the body, regardless of its place or status in the universe—that is, irrespective of whether it is "in space," is "falling," or has "fallen"; accordingly, this term obviates the need for that obsolescent expression *meteoroid*[3] and makes one word serve, both for the meteorite in space and for each of the components into which the falling meteorite may disintegrate and which may become "complete individuals," technically so-called, or "fallen meteorites." We do not know and we may never know, save, possibly, in very exceptional cases, whether the disjunct members of a multiple fall or meteoritic shower are produced by the disruption of a *single* falling meteorite or whether they are the remains of individual members of a *swarm* of meteorites that entered the Earth's atmosphere as such—or both.[4] Lacking any information on the matter, we are, then, justified in denominating each *complete* specimen of a multiple fall, a "fallen meteorite."

It is interesting to compare the traditional dictionary definitions of the terms *meteorite, meteoroid,* and *meteor* with the present definitions of these terms. The comparison leads to the following table:

Dictionary Term		*Present Term*
Meteorite	=	Fallen meteorite on Earth
Meteoroid	=	Meteorite in space
Meteor	=	Incandescent falling meteorite (or incandescent meteorite in flight) in the Earth's atmosphere

According even to the lexicon definition, the word *meteor* does *not* mean a "meteorite in space" (a sense in which the term *meteor* is so often misused, especially by writers of textbooks!), but rather an "incandescent falling meteorite in the Earth's atmosphere."[5]

NOTES AND REFERENCES

[1] The specific meaning to be inferred from the unqualified term *meteorite* is nearly always apparent from the context.

[2] As defined *both* ways, *ante.*

[3] Which means only a "meteorite in space," in accordance with definition (1), *ante.*

[4] *Cf.* the writer, *C.S.R.M.*, **1**, No. 3, 2-3; *P. A.*, **45**, 47, 1937.

[5] *Cf.*, however, the definition in the third paragraph of this paper.

President of the Society: LINCOLN LA PAZ, Department of Mathematics, Ohio State University, Columbus 10

Secretary of the Society: C. H. CLEMINSHAW, Griffith Observatory, P. O. Box 9866, Los Feliz Station, Los Angeles 27, California

Comet Notes

By G. VAN BIESBROECK

Comets observable in ordinary instruments have been unusually lacking these last months. None are known to be observable at this time.

The search for the expected periodic comet, PONS WINNECKE, has not been successful so far. On the other hand PERIODIC COMET 1943 *a* (OTERMA) has now been recorded again in the morning sky, quite close to the position predicted by P. Herget. It was found on two plates taken by the writer with the 82-inch reflector of the McDonald Observatory, Fort Davis (Texas), in the position:

1945 APRIL 14.462 U.T.

Right ascension	$19^h\,54^m\,15^s$	1950
Declination	$-17°\,36'$	

The comet appears as a round nebulosity of 6″ diameter and of magnitude 17. Herget's prediction requires only the small correction of -0^m2 in right ascension, with no correction in declination. It was based on his elements:

EPOCH 1943 OCT. 3.0 U.T. $M = 50°\,89373$
MEAN MOTION $0°.1249872$ (PERIOD 7.8857 YEARS)

Semi-major axis	3.961807	A.U.
Eccentricity	0°.1444246	
Inclination	4°.98994	1950
Ascending node	155°.17075	1950
Perihelion to node	354°.80579	1950

The remarkably small eccentricity and inclination make this a distinctly planetary orbit (see POPULAR ASTRONOMY, **52**, 154, 1944). The fuzzy appearance cannot, however, leave any doubt as to the non-planetary character of this small object.

Fort Davis, Texas, April 14, 1945.

General Notes

Miss Martha E. Stahr, now holding the Alice Freeman Palmer fellowship from Wellesley College and engaged in research at the Lick Observatory, has been appointed instructor in astronomy at Wellesley College.

Professor W. J. Luyten, of the University of Minnesota, recently spent two weeks at the University of Arizona, using the 36-inch reflector of the Steward Observatory, to work on his program of White Dwarfs, in collaboration with Dr. P. D. Jose, of the University of Arizona. (*Science*, April 13, 1945.)

Dr. F. J. Neubauer, Associate Astronomer and Librarian in the Lick Observatory of the University of California, and President, this year, of the Astronomical Society of the Pacific, will teach one course in elementary astronomy and another course in navigation and nautical astronomy in the Summer Session of the University of California, Los Angeles, which will be held from July 2 to August 10, 1945.

Dr. Helen W. Dodson, instructor in astronomy at Wellesley College from 1933 to 1937 and assistant professor since 1937, has been appointed associate pro-

fessor of astronomy and mathematics at Goucher College. Since December, 1943, Miss Dodson has been on leave from Wellesley and occupied with war work at the Radiation Laboratory of the Massachusetts Institute of Technology.

Dr. Karel Hujer of Michigan State College has been invited to conduct a seminar related to his field of Astronomy at a summer conference under sponsorship of the Federal Council of the Churches of Christ in America at Lake Winnipesaukee, N. H., July 28-August 7. Dr. Hujer will be lecturing there with Dr. E. Stanley Jones. Also, commencing May 4, Dr. Hujer will broadcast every Friday at 6:00 P.M., CWT, over Michigan State College Radio Station WKAR, 870 kilocycles, on the subject "Evenings Under the Stars."

Dr. Harlow Shapley receives the Franklin Medal

Founded in 1914, the Franklin medal, highest honor of The Franklin Institute, has been awarded annually to "those workers in physical science or technology, without regard to country, whose efforts have done most to advance a knowledge of physical science or its applications." Thomas A. Edison, Guglielmo Marconi, Charles Fabry, Pieter Zeeman, James H. Jeans, Orville Wright,

DR. HARLOW SHAPLEY

Albert Einstein, Charles F. Kettering, Peter Kapitza, and William David Coolidge are among the former recipients of the medal. The face of the medal carries a medallion of Benjamin Franklin done from the Thomas Sully portrait owned by The Franklin Institute.

Dr. Shapley receives the Franklin medal this year "in consideration of his many valuable contributions to the science of astronomy, and especially of his work in the measurement of the vast distances necessary for the determination of the nature and extent of our galaxy, as well as those of other galaxies external

to ours."

Dr. Shapley's contributions to astronomy have covered a broad field and perhaps his greatest work has been in plumbing the depths of the universe and in determining the dimensions and distributions of globular clusters, galaxies, and super-galaxies. For this purpose his principal measuring rod has been the statistical relationship between luminosity and period of certain variable stars. Thus when the period of a variable has been determined its intrinsic luminosity can be found. This, together with the star's apparent brightness, yields a measure of distance. He has worked out also relationships between intrinsic brightness and color, or spectral type, of giant stars in star clusters.

His versatility is evidenced by his studies of eclipsing variables, the photometric measurement of North Polar stars to the twentieth magnitude, the nature and the cause of variation of the Cepheid type, the spectra and periods of Cepheid variables, the measurement of the dimensions of our own galaxy, the nature and distribution of external galaxies, the density of matter in space, and the structure of the universe.

Besides his many contributions to astronomy, Dr. Shapley is a great lecturer and has published several technical papers on entomology and geology. His most recent work deals with galaxies, star clusters, variable stars, and the astronomical dating of the creation of the earth's crust.

Dr. Shapley is now president of the American Astronomical Society, of the National Society of the Sigma Xi, and Science Clubs of America, and chairman of the Board of Science Service, the Worcester Foundation for Experimental Biology, and the World Wide Broadcasting Foundation. He is a former president of the American Academy of Arts and Sciences.

The Herschel Astronomical Club of Fall River held its fourth meeting of the 1944-45 season recently. The principal speaker was Lieut. Eugene T. Oliver, USN, who gave an illustrated talk on the subject, "Elements of Navigation." Mr. Samuel R. Parks, supervisor of education in Fall River, was elected president of the club for the sixth successive year.

FRANK J. KELLY.

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Sacramento Valley Astronomical Society

An amateur astronomical society was organized on March 5 under the above name. It has an initial membership of some forty, and will hold meetings in Sacramento Junior College. The society intends to have frequent out-of-door meetings, using the several telescopes available for observations.

The officers of the new society are: Charles A. Fogus, President; George C. Kimber, Vice-President; Samuel J. Smyth, Secretary-treasurer. These officers and Paul Mason and Dr. Robert E. Smith constitute a Board of Directors. We hope to be able to publish from time to time the reports of the activities of this new organization.

The Cleveland Astronomical Society

Dr. Peter van de Kamp of the Sproul Observatory, Swarthmore College, lectured on "The Nearest Stars" at the March meeting of the Cleveland Astronomical Society. He limited his discussion to those stars which are within 16 light years of the sun. These stars are discovered by means of their large proper

motions. Barnard's star, for example, moves a moon's diameter in 170 years. Its motion can be detected on two plates three nights apart.

There are probably about 50 of these near stars. The bright ones are known but there may be more faint ones which are difficult to find. Their range in luminosity is very great, of the order of 1 to 50,000. Only four are brighter than the sun. Sirius is 30 times as bright as the sun while Wolf 359 is 50,000 times fainter than the sun. The majority of these stars are red dwarfs. Their range in mass is very small, about 1 to 20.

The speaker mentioned the theories used to explain what keeps the stars shining. He also mentioned the frequency of double stars. Of the five nearest stars four are known to be double or multiple. These questions were asked by the speaker. Is the sun exceptional in its singleness? Was it perhaps once double? He suggested that the search of the future may be for single stars as they may be the exception rather than the rule.

The Ohio Neighborhood Astronomical Group met the following afternoon, March 3, and were addressed by Dr. van de Kamp on "Unresolved Astrometric Binaries." These are the binaries which cannot be detected by ordinary visual or photographic methods or by means of the spectrograph. They are discovered by perturbations in their proper motions and much can be learned from a study of these perturbations. Their determination is therefore limited to those near stars whose proper motion may be accurately determined. This interesting lecture was followed by a dinner.

HENRY F. DONNER.

Western Reserve University, Cleveland 6, Ohio.

The Cleveland Astronomical Society

At the April meeting of the Cleveland Astronomical Society the audience was privileged to hear an interesting and well-illustrated lecture on "The Revelations of the Spectroscope" by Dr. C. K. Seyfert of the Warner and Swasey Observatory, Cleveland, Ohio.

Dr. Seyfert described the slit spectroscope and demonstrated how spectra are obtained. He described the different types of spectra resulting from varying conditions at and surrounding the light source and showed how these spectra may be used to determine the elements in the source of light and its surrounding atmosphere.

With the slit spectroscope only one star spectrum can be photographed at a time. With the objective prism as used on the 36-inch Schmidt-type telescope at the Warner and Swasey Observatory the spectra of a hundred or more stars may be photographed at the same time which is a great time saver. The high light of the evening was probably two kodachrome slides taken with this telescope. One showed the spectrum of Arcturus in color and another showed the individual spectra, in color, of a group of stars, the first of their kind ever to be made.

The speaker explained how the spectroscope is used to determine the elements in the atmospheres surrounding stars and the compounds in the cooler atmospheres surrounding some of the planets. Stars are classified according to their spectral type. More accurate colors may be determined from relative intensities of different parts of the continuous spectrum. From these colors temperatures may be determined which may also be obtained from relative intensities of the dark lines in different parts of the spectrum. By means of the temperature-

luminosity curve the intrinsic luminosity of a star may be obtained if the temperature is known. With the intrinsic brightness and apparent brightness known the distance may be computed. In the use of the temperature-luminosity curve it is usually necessary to know if the star is a giant or dwarf. The difference is detected on plates taken with the Schmidt-type telescope by a decrease in spectrum width in a portion of the spectrum for the giants but not for the dwarfs.

Other uses of the spectroscope described were the determination of star velocities in the line of sight by the amount of shift of the dark lines of the spectrum, discovery and measurement of spectroscopic binaries by the periodic doubling of the spectrum lines, measuring the movement of galaxies which all seem to be traveling away from us with speeds that increase with their distance from us suggesting an expanding universe.

After the lecture the members had the opportunity of seeing stellar spectra through the Schmidt-type telescope and Saturn and Jupiter through the refractor.

HENRY F. DONNER.

Western Reserve University, Cleveland 6, Ohio.

Prediction of Next Sun-spot Maximum

The sun-spot cycle that started in 1934 came to an end in 1943 and we are now approaching the years of maximum solar activity of the cycle that started about two years ago.

The following table of the sun-spot cycles for the century 1843-1943 shows the Wolf yearly peak maximum numbers and the monthly peak maximum numbers:

Cycle	Wolf Yearly Peak Maximum Numbers	Wolf Monthly Peak Maximum Numbers	
1843-1856	1848-124.3	October,	1847-180.4
1856-1867	1860- 95.7	July,	1860-116.7
1867-1879	1870-139.1	May,	1870-176.0
1879-1889	1883- 63.7	April,	1882- 95.8
1889-1901	1893- 84.9	August,	1893-129.2
1901-1913	1905- 63.5	February,	1907-108.2
1913-1923	1917-103.9	August,	1917-154.5
1923-1934	1928- 77.8	December,	1929-108.0
1934-1943	1937-118.8	July,	1938-165.3

A study of these nine cycles indicates clearly that the coming maximum will be considerably lower than that of the preceding adjacent cycle.

It will be observed that there is a regular alternation of the height of maximum in successive cycles consistent for the century. This argues strongly, almost conclusively, for a much lower peak maximum for the present cycle than for the cycle of 1934-1943.

The highest maximum Wolf number for the alternate cycles having low yearly maxima is that of 1860-95.7 and the lowest that of 1905-63.5. A peak maximum somewhere between the two numbers may be predicted.

RALPH C. LINDER.

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Sun-spot Observations

On page 519 of the December (1944) issue is to be found a statement regarding sun-spot observations to be made by members of the A.A.V.S.O. The need for stimulating this phase of astronomical work was felt because of the reduced

number of workers in astronomy as a result of the demands of war. Already reports have been sent in which have been found useful by the Department of Terrestrial Magnetism, Washington, D. C. Those in charge of the promotion of this work are eager to enlist the cooperation of colleges and universities in which such work can be made a part of existing courses. Further information may be secured from Mr. Neal J. Heines (chairman) 560 Broadway, Paterson 4, N. J.

The Occultation of Venus

The occultation of Venus, on the evening of December 18, 1944, combined a rare sight of beauty and an excellent chance to time the immersion, as viewed in Miami, Florida.

Using a three-inch reflector and a low-power eyepiece, it was indeed a sight of beauty, witnessed by numerous friends prior to the planet's sudden disappearance.

The writer, a navigator for the Air Transport Command, set his 21 jewel Hamilton chronometer to the exact second, using a time tick obtained from WWV, the U.S. Bureau of Standards. A stop watch, of known rate, was started at 0100 G.C.T. or 9:00 P.M. (EWT) and stopped at the exact moment of immersion.

The observation was made at Longitude 80° 18′ West, Latitude 25° 44′ North.

The time of immersion as viewed thru a 210 power eyepiece at the above stated location was 01101.1 GCT or 9:10:01.1 EWT. (P.M.)

J. WESLEY SIMPSON.

1529 S.W. 7th Street, Miami 35, Florida, March 10, 1945.

Book Reviews

Physics of the 20th Century. by Pascual Jordan. (Philosophical Library, Inc., 15 E. 40th St., New York 16, N. Y. $4.00.)

The laymen who has but a rudimentary knowledge of science and philosophy owes a debt of gratitude to Pascual Jordan for this interesting and informative book, and to Eleanor Oshry for a clear and adequate translation. Also the student of science, philosophy, or religion may spend profitable hours in reading and gauging the import of the wide range of scientific advance in physics here so convincingly portrayed.

The fact that conclusions reached by positivistic science up to recent times, supporting a mechanistic and fatalistic world-view, have been disproved in the realm of microphysics, and hence modified throughout, indicates the inadequacy of positivism for philosophy and religion. Nevertheless, "Physics of the 20th Century" is clearer and more convincing because Jordan has confined himself in the main to the findings of positivist physics, only incidentally and briefly pointing to consequences for philosophy and religion that may profitably be explored. As he indicates, these are particularly found in relation to living organisms such as man. Modern discoveries of indeterminate behavior in the microphysical realm are of peculiar interest to psychology and religion, as these face the problems of erroneous and idealistic behavior and human freedom in general. Theology may also find here a point of contact for the creative and directive activity of God.

The layman or student, because he is not burdened with details, is likely to

gain a more comprehensive understanding of the laws and limits of causality, of the unrealized possibilities of science which is so dependable in the macrocosm and yet so flexible in the microcosm, and of the resulting implications for ethical and religious as well as for materialistic advance.

Carleton College. KENNETH W. WEGNER.

The Comet of 1577, Its Place in the History of Astronomy, by C. Doris Hellman. (New York-Columbia University Press 1944. Price $6.00.)

The comet which appeared in November, 1577, is of great historic interest. Through its observation Tycho Brahe was the first to prove that comets are not terrestrial phenomena and that they move in regions far beyond the limits of our atmosphere and the distance of the moon.

It was a bright object and caused a flood of writings both descriptive and speculative. For her 488-page volume Dr. C. Doris Hellman has painstakingly compiled all this literature and presented the various ideas concerning the nature of comets that were current from antiquity to the time of the 1577 apparition. Even at this date most writers leaned heavily on Aristotle's authority to repeat that comets were purely atmospheric phenomena and to correlate their appearance with climatic conditions and terrestrial events in general. The author presents these opinions with an exceptional amount of erudition. To the modern reader it is depressing to follow the story of all those reiterated fallacies. It is quite a relief to see how at last through a simple observation all these phantasies are pushed in the background. The author delights in biographical notes concerning the various writers and in listing all their bibliographical contributions even those that are not related to the subject of the book. The subject could not have been treated in a more exhaustive manner. G.V.B.

Publications Received.—The publishers of POPULAR ASTRONOMY hereby acknowledge receipt of the following named publications and express their great appreciation of the courtesy shown on the part of those who have sent them.

Contributions of the Mount Wilson Observatory:

No. 693. "Orbit of the Ninth Satellite of Jupiter," by Seth B. Nicholson.

No. 694. "Effect of Temperature on the Wave Length of a Transmission Band of the Interference Polarizing Monochromator," by Edison Pettit.

No. 695. "Doubly Ionized Rare Earths in α^2 Canum Venaticorum," by P. Swings.

No. 696-697. "The Resolution of Messier 32, NGC 205, and the Central Region of the Andromeda Nebula."

"NGC 147 and NGC 185, Two New Members of the Local Group of Galaxies," by W. Baade.

No. 698-699. "Regression Lines and the Functional Relation," by Frederick H. Seares.

"Relation Between Color Index and Effective Wave Length from the Observations of Hertzsprung and Vanderlinden," by Frederick H. Seares and Mary C. Joyner.